THE GROUNDWORK OF LOGIC

PART I

PRINCIPLES OF DEDUCTION

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PART I

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TO

MY ESTEEMED TEACHER,

Mr. A. K. SHAH,

PRINCIPAL, CALCUTTA BLIND SCHOOL,
who guided me through my life's greatest crisis.

PREFACE

Men, as rational beings, will never be able to dispense with the study of the forms of thought. The science of Logic will always form an important branch of study in every university which intends to impart liberal education to its students. Logic, like other sciences, has developed slowly and gradually, and though most of its problems are as old as Aristotle, their solution and treatment are no longer the same as in ancient times. The Scholastic logicians wrongly supposed that logic is a stereotyped science, and tried to bring all processes of thought under definite formulæ propounded by Aristotle. Since the time of Bacon there has been a change in the outlook of logic. Bacon insisted that the matter of thought should be as much studied as its forms, and from his time greater interest began to be taken in observation of and experiment upon facts. In England Mill raised induction to a definite science, but he was one-sided, inasmuch as he supposed that the only form of inference is induction, to the exclusion of ratiocination. Inference, according to him, is always a passage from particulars to particulars. In this way two schools of logicians, *viz.* the advocates of formal logic and the advocates of material logic, were drawn into hostile camps, each group differing fundamentally from the other. Formal logicians claimed that logic is always formal, while the other school held that every inference is inductive, and that the study of the forms of thought alone can never lead to the attainment

of truth. Thus one school studied forms of thought alone while the other claimed to study its matter. Formal logicians tried to reduce induction to deduction, while advocates of material logic attempted to reduce deduction to induction.

But this long-drawn controversy is now over, and recent logicians such as Bradley, Bosanquet, Johnson, Russell etc. have shown beyond doubt that thought is organic, that it is a whole consisting of parts which are in intimate relation with each other, and that induction and deduction are not two distinct processes but two complementary aspects of thought. It has also been clearly shown that conception, judgment and inference are not distinct processes of thought, but that conception naturally passes into judgment and judgment into inference. Elements of inference are present in both judgment and conception, and logic should not only devote itself to the study of the conditions of valid thinking, but should also point out how elementary forms of thought gradually pass into developed and complex forms. The study of judgment enables us to understand the nature of inference no less than the study of inference itself. The intimate relation existing between different parts of logic and different forms of inference was not clearly perceived by ancient logicians, and it is a definite contribution of Hegel, Bradley, Bosanquet and others to the science of logic that they have shown that it is an organic and fluid science. Recent logicians, therefore, devote greater attention to the study of judgment or proposition, because they rightly hold that a clear

understanding of the forms of judgment enables us to understand the forms of inference as well.

In discussing the problems of logic, I have tried to treat of every problem of traditional logic in the light of present-day discussions. The problem of the proposition being central, I have devoted more pages to it than is usual in elementary books. There is no doubt that without a clear understanding of the forms of propositions it is impossible to possess an insight into the nature of the forms of inference.

This book is mainly intended for the Intermediate students of Indian Universities, but I hope it will be useful also to those who want to study philosophy and higher logic afterwards. My experience as a teacher of logic for several years has convinced me that it is very unfortunate that Intermediate students learn to regard logic as a stereotyped science, and I hope that this book will to some extent dispel this illusion from their minds. Some portions of the book may appear difficult to elementary students, *e.g.* the first two paragraphs of the chapter on Terms, the first chapter of the book on Propositions, the discussion of the general nature of Inference and of Syllogism, etc. ; but I have tried my best to treat them as lucidly as possible, and I feel sure that if students take some pains, it will not be impossible for them to understand these topics. Even if they cannot grasp the whole of these portions, it will be something if they understand the salient points of these topics, which I believe they will do. I have tried to discuss every problem from different points of view, and have tried

to make the book as comprehensive and exhaustive as possible within the limits prescribed. I have quoted authorities wherever possible to clarify every problem, and to acquaint students with different opinions. Whenever I have differed from the authorities I quote, I have stated the reasons.

I am very much indebted to all those writers on logic to whose writings I have referred from time to time in the body of the book, but I may specially mention the names of Mill, Keynes, Bradley, Bosanquet, and Joseph. In this volume I have treated of the problems of deduction only, and shall deal with the principles of induction in another volume. Space did not permit me to provide exercises in this volume, but I expect to do so in the appendix to the next volume. For the convenience of students I have devoted the last chapter of this volume to the discussion of formal fallacies. I shall be grateful to all those readers who will be good enough to send suggestions to me to enable me to make improvements upon this book.

It is impossible for me to express my sense of gratitude to my revered teacher, Prof. H. H. Crabtree, M.A. (Oxon.), of St. Paul's College, Calcutta, without whose assistance it would have been impossible for me to publish this book now. He read through the whole of my manuscript, and his suggestions enabled me to make several very valuable improvements and to get rid of some of the mistakes which would otherwise have remained unnoticed. He has read all the proofs of my book very carefully, and it is due to his sincere and kind

help that I have been able to get rid of printing mistakes to a large extent. Prof. Crabtree has also been good enough to prepare all the diagrams which the book contains, with the explanations of them, and the table of contents provided has also been prepared by him. I cannot but express my sincere thanks to Babu Sisulal Banerjea, B.A., without whose sincere co-operation and assistance it would have been impossible for me to prepare the manuscript. He has had to labour unceasingly along with me day after day in the preparation of this book.

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PRINCIPAL WORKS REFERRED TO.

The following works are usually referred to simply by the name of the author :

B. BOSANQUET	Essentials of Logic.
F. H. BRADLEY	The Principles of Logic.
W. E. JOHNSON	Logic.
H. W. B. JOSEPH	An Introduction to Logic.
J. N. KEYNES	Studies and Exercises in Formal Logic.
J. S. MILL	A System of Logic.
J. WELTON	Manual of Logic.
J. WELTON & A. J. MONAHAN ...	An Intermediate Logic, 3rd edition revised by E. M. Whetnall.

THE GROUNDWORK OF LOGIC

PART I

PRINCIPLES OF DEDUCTION

INTRODUCTION

CHAPTER I

DEFINITION, NATURE AND SCOPE OF LOGIC

Knowledge and its Sources

Knowledge is progressive and since men often err, it is necessary to investigate the conditions of valid thinking.

Without thinking no knowledge is possible. Men as rational beings cannot do without thinking just as they cannot live without food. But the knowledge of one generation is often rejected by subsequent generations. So it is true to say that the path along which knowledge progresses is strewn with the wreckage of exploded theories. We thus find that most sciences give up old beliefs and accept new ones as the result of progressive investigation and enlightenment. The new astronomy is no longer the old astronomy of the Greeks nor is the modern physics the same as the crude physics of antiquity. So men have become modest and they no longer think that knowledge is stereotyped, fixed and unprogressive. They have therefore ceased to suppose that the accepted knowledge of one generation has a dictatorial function over all times to come. Though this is true, men have extended their mental activity to the

conquest of the forces of Nature and are more and more mastering the secrets of the world. In spite of this, men cannot avoid errors altogether, and since failures are inevitable for men, it is necessary to investigate the conditions of right thinking.

Knowledge begins with thinking. Thought may be either perceptual or conceptual. Distinction between thought and sensation or sense-experience.

An analysis of knowledge gives us two main types, *viz.* perceptual or immediate knowledge and inferential or mediate knowledge. Though sense-experiences are supposed by many to be the basis of knowledge, they do not themselves constitute knowledge. They are rightly regarded as the raw materials of knowledge. Knowledge begins only when we interpret these experiences to construct the world of thought. (Perception is the process by which particular sense-experiences are interpreted and objectified.) Suppose I am sitting in the field one evening and various shapes, sounds, odours etc. are leaving their impressions on my mind but I remain passive. In this case there is no thinking and therefore no knowledge, though the materials out of which knowledge may be constructed are given and present in my consciousness. Suppose I give up my passive attitude and become active and begin to interpret my sense-experiences. I say to myself, This is a flower, A cuckoo is singing, This odour is sweet, That tree is large, etc., and as soon as I thus interpret my particular experiences I think about them and have perceptual knowledge. Perceptual knowledge is the knowledge of individual objects, and is distinguished from conceptual or

general knowledge. Conception may be roughly defined for the present as the knowledge of the common attributes of a class. Thus, 'this man is tall' gives us perceptual knowledge, because this proposition refers to one particular man, while the propositions 'all material bodies gravitate,' 'all men are mortal,' give us knowledge of classes, not of individuals, and such knowledge is conceptual. Mr. W. E. Johnson rightly remarks that thought includes both conception and perception, since it aims at interpreting the universe. Perception, like conceptual judgment, involves activity and is controlled by the purpose of attaining truth. This purpose is the characteristic mark of thought, which may be defined as 'mental activity controlled by a single purpose, the attainment of truth.'

Our thought is true when it agrees with the actual world.

When is our thought true? or in other words, what constitutes truth? Whenever we think, we think of some object, whether our thought is perceptual or conceptual. Thought begins with judgment. The judgment 'This flower is red' has objective reference, and is true if it agrees with the nature of the object to which it refers, that is, if this flower is really red. But this judgment or thought would be false if the flower were green or of some other colour, or if the object, to which the judgment refers, were something different from a flower. Similarly the judgment 'all material bodies gravitate' will be true if in the actual world material bodies without exception do gravitate. So we may say that our thought is true when it corresponds or agrees with some

aspect of reality, that is, with the actual world. We should remember that truth and falsity can be predicated only of thought. Hereafter we shall distinguish between formal truth and material truth, when we deal with formal logic and material logic.

Immediate knowledge. Perception involves elements of implicit or unconscious inference.

We may now distinguish between perceptual or immediate knowledge, and conceptual or mediate knowledge. When my knowledge of a certain object is direct, that is, when it is not acquired through the knowledge of some other object, it is immediate. Thus the knowledge of sensuous objects, *e.g.* 'this man is tall,' 'this drink is sweet,' etc. is immediate and perceptual. But thinkers recognise instances of immediate or intuitional knowledge which is not perceptual, *e.g.* the knowledge that A is A, that every event has a cause, etc. The intuitional school of philosophy recognises the existence of immediate knowledge which is not perceptual while empirical philosophers hold that perceptual knowledge alone may be regarded as immediate and direct. But even perceptual knowledge cannot be regarded as purely immediate, that is, free from all elements of inference. Take the judgment 'this man is tall.' Though this judgment is perceptual, it has elements of inference in it, because without previous knowledge of man and of tallness I could not have the judgment. To recognise that the object of my perceptions is a tall man requires previous knowledge and therefore it is inferential, though in this case inference is implicit or unconscious. Inferential knowledge is commonly regarded as mediate knowledge because such knowledge is acquired through the mediation of the

knowledge of other objects, that is, indirectly. All conceptual knowledge is inferential. I cannot have the knowledge that 'all men are mortal' without having the previous experience of particular men dying. Similarly we cannot have the judgment that 'water boils at 212° Fahr.' without having the experience of particular cases of water boiling at that temperature. Again when I pass to the conclusion 'Socrates is mortal' from the premises 'all men are mortal' and 'Socrates is a man', my knowledge is mediate or indirect because it is acquired through the knowledge of the two given premises. We have already remarked that some conceptual judgments are regarded by the intuitional school of philosophy as immediate, *e.g.* A cannot be not-A, A is either B or not-B, etc. Knowledge that is acquired through comparison of similars is also inferential, *e.g.* 'the whiteness of snow is like the whiteness of milk'. Inferential knowledge is commonly regarded as mediate.

Authoritative knowledge being inferential the main sources of knowledge are perception and inference.

Besides the two main sources of knowledge, *viz.* perception and inference, another source is recognised, *viz.* authority or verbal testimony. But knowledge acquired through authority cannot be properly regarded as derived from a distinct source, being really inferential though it has some appearance of directness. A large part of our knowledge is acquired through authority and all the knowledge of the past to which we have access has been preserved for us through language which is the instrument of thought. (The knowledge that we acquire through the study of great

scientists, historians and philosophers appears to be immediate and direct but really such knowledge is inferential. I accept as true the views of those authors and the testimony of those persons in whom I have confidence or who can stand criticism. Even if we regard authoritative knowledge as midway between direct or immediate and indirect or mediate knowledge, we have but two main sources of knowledge, *viz.* perception and inference. We cannot here enter into the controversy whether there is the possibility of general yet immediate knowledge or intuitional knowledge of general truths, as is advocated by the intuitional school of philosophy Further, if it is true that even perception involves inference, then the only source of knowledge becomes inference, unconscious and conscious.

Definition of Logic

Definition of Logic Scientific knowledge is universal, necessary, definite and accurate and is not the same as popular knowledge. Explanation of the definition which appears to us to be satisfactory.

Before examining some of the outstanding definitions of logic we may at the outset formulate and explain the definition of logic that appears to us to be satisfactory. Logic has rightly been defined as the science of the principles of valid thought, or it may rather be defined as the science of valid thought, because every science studies principles and this is no peculiarity of the science of logic. Every science systematically studies some branch of the phenomenal world or the world as it appears to us. Thus scientific knowledge is different from popular knowledge, which applies itself to the study of

particular phenomena without trying to find out the interrelation between them. Scientific knowledge is universal, necessary, definite and accurate. Knowledge that is universal and necessary is always and everywhere true and is true for all men. It is not restricted by considerations of time and space Further, every science studies laws or principles which hold good uniformly of a number of objects between which laws are the connecting links. Thus, when we have discovered laws regarding the definite groups of phenomena which particular sciences study, we can discover in addition that the different groups themselves are connected with one another, that is, are interrelated and not isolated. Thus the sciences try to find out causal or other such uniform relations between phenomena. Scientific knowledge is very useful because by studying a few fundamental principles we can gain knowledge of a very large number of objects. The things of the world are innumerable and indefinite and no man can know each of them, but the study of them is reduced by science to the study of some fundamental principles which it is possible for the human mind to know in a general way. It is not possible for us to observe whether every material body gravitates or not, but we can have the knowledge of each of them if we have access to the principle or law recognised by physics that all material bodies gravitate. Thus botany studies the laws according to which different plants behave, originate and grow, and reduces to a system the plant world. It is not possible for us to study all plants individually, for they are numberless, but we can have knowledge of all of them if we know the principles which govern the plant-kingdom.

Similarly we can have general knowledge of the behaviour of the planets if we have access to the science of astronomy, which systematically studies the principles according to which they move. Similarly geometry studies spatial relations and physics and chemistry the relations existing between physical objects and their properties and so on. Just as every other science studies laws or principles regarding its own subject-matter, so also logic studies the principles of its own subject-matter, *viz.* thought. Just as botany is the science of plants and astronomy of heavenly bodies, logic studies thought. We have already seen that thought may be either perceptual or conceptual and logic studies both these aspects of thought. Thought further may mean either the process of thinking or the product of thinking. Logic, though it is mainly concerned with the process of thinking, studies also the product of thinking. But logic does not study thought as it is, but thought as it ought to be in order to be valid or true. We have already seen that thought is valid when it agrees with the real world, that is, when it is accepted by some content of reality. Men think correctly or incorrectly, and logic by examining thought finds out the conditions of valid thinking. So to define logic as the science of thought, as Joseph does, is inadequate, since logic does not study all thought but only the conditions of valid thinking. Johnson defines logic as "analysis and criticism of thought," but this definition is also too wide because it does not clearly bring out the nature of logic and does not even tell us that it is a science. Logic no doubt criticises thought but its object is to bring out the principles of valid thinking; besides the

significance of the word 'thought' is more or less vague.

Logic, Ethics & Aesthetics, the three normative sciences. Their nature explained.

We may further explain the nature of logic, which is an ideal or normative science, by comparing it with the two sciences recognised as normative, *viz.* ethics and aesthetics. The ideal of logic is truth, that of ethics goodness and that of aesthetics beauty. Logic is the science of valid thought, ethics of right conduct and aesthetics of correct taste. Logic is concerned with thought, ethics with volition or will and aesthetics with feeling. The ideal of logic however, *viz.* truth, is more clear and definite than the ideal either of ethics, which is the good life, or of aesthetics, which is beauty or proportion. Besides, ethics and aesthetics are closer to each other than either of them is to logic.

Logic is a science and not an art. Its aim is not to lay down precepts to make men reason well. Logic however may properly be regarded as a practical science.

We may now consider whether logic besides being a science is also an art. The Port Royal logic defines logic as "the Art of Reasoning," while Whately defines it as "the science, as well as the art, of reasoning." Many other logicians hold that logic is an art. Without criticising the above definitions in detail, we may ask ourselves whether logic can properly be regarded as an art. According to Joseph art may mean "practical skill in doing a thing or theoretical knowledge of the way in which it is best done." In the first sense cooking, carpentry etc. are arts, in the second sense navigation, music etc. may be regarded as arts. But if we interpret art in the

second sense, it requires the help of other sciences. Thus navigation cannot be successful without some knowledge of astronomy, mechanics, meteorology, physics, mathematics etc. In this sense logic is regarded as an art because it is supposed that the knowledge of the principles of thought enables men to reason correctly and to avoid fallacies and to detect errors. In this sense politics and ethics were regarded by Aristotle as arts, because according to him a knowledge of the principles of the state would enable a man to act successfully for the good of the state. Similarly the knowledge of the principles of good conduct would make a man act rightly. An art, therefore, is a body of precepts for performing some work. Thus logic is divided into theoretical logic and practical logic. We do not deny that logic is practical, for it disciplines the mind well and a disciplined mind can think well and act well. But logic should not on that account be regarded as an art. As the aim of ethics is to define the ideal of good conduct, so the aim of logic is to define the ideal of truth. Logic no more teaches men to reason well and to avoid fallacies than ethics teaches them to act rightly. Logicians often argue falsely and ordinary men unacquainted with logic often reason correctly. Men thought rightly or wrongly and acted rightly or wrongly before the sciences of logic and ethics were recognised. So Locke says, "God has not been so sparing to men, to make them barely two-legged creatures, and left it to Aristotle to make them rational." The proper function of light is to dispel darkness, and when darkness is removed, men can move about easily without falling. But it is not the function of light to enable men to walk without falling. Similarly though the knowledge of the

principles of logic may enable men very often to avoid fallacies and to reason correctly, its aim is not to lay down precepts to make them reason correctly. But since it is the right of logic to examine the conclusions of every science and since every science has to follow logical method, it is undeniable that logic is a practical science. Though Mill recognises the theoretical aspect of logic, he like many others regards it as a practical science. So he defines logic as "the science which treats of the operations of the human understanding in the pursuit of truth," or as "the science of the operations of the understanding which are subservient to the estimation of evidence." Mill, however, over-emphasises the practical aspect of logic.

Logic cannot be regarded as a formal science pure and simple.

Formal logicians like Kant, Mansel, Ward, Hamilton etc. regard logic as the science of the formal principles of thought. They make a distinction between form and matter and suppose that logic studies the form of thought only and not its matter. Let us see what they mean by form. By form we mean that which is the same in many individuals materially different. Thus coins though materially different have the same form. Similarly different horses, different men etc. have the same form though individual horses and men differ materially. Aristotle rightly points out that forms exist in individuals and there cannot be any form without matter. Why should we specially regard logic alone as formal? Every science has to study forms or general principles existing in individual instances. Logic does not study the formal principles of thought alone, but the conditions of valid thinking.

Thought is valid only when it corresponds with the nature of the real world. No doubt it studies forms of thought, but it studies the matter of thought as well. Besides, if we regard logic merely as the science of the formal principles of thought, it may suggest that there may be form without matter and science without principles. But since truth has always objective reference, logic, though it studies the forms of thought, cannot ignore the matter of thought. Even formal or deductive logic cannot altogether ignore the consideration of the material universe, as we shall hereafter find, and induction, though it can be formally treated up to a certain point, has always to base its conclusions upon the observation of facts.

Since truth has always objective reference, logic cannot ignore the matter of thought.

Logic and Language

We have seen that logic is the science of valid thought. Thought has always an objective reference, that is, whenever we think, we think of something. An individual mind can think of the material world, of other minds, and can reflect upon itself as well. So the object of thought may be the states of one's own consciousness or something other than the states of one's own consciousness, whether it be some other mind or some material object external to himself. In a word the object of thought is the whole universe, consisting of objects both real and imaginary. We can think of this world as a unity or can reflect upon some aspect of it. By our

Nature of Logic and why it is related to language.

thought we may be said to construct our world, for our world to us is what our knowledge makes it to be. In search of truth we try by means of our thought or judgment to interpret this world rightly. But we often think wrongly, and therefore logic, by criticising thought, tries to systematise the principles of valid thinking. Thought which is true must be universal and necessary, that is, true to all men at all times; what is true at all can never be otherwise. Thus logic is concerned with universal thought and not with individual idiosyncrasies or peculiarities. It studies the thoughts of normally constituted minds. Just as the universe is one, so there is unity in the different processes of thought. As logic studies universal thought, it requires the help of a universal instrument to express it. This universal instrument of thought is language.

The different ways in which language helps thinking.

Expressive gestures, such as asking a person to come by the movement of the hand, are the elementary forms of language. But logic is not concerned with such a language or instrument of thought. It is related to conventional language, that is, to written or spoken words, because by means of such language we can discriminate between different ideas. Language is as universal as thought itself. Language aids thought in various ways. Though thought is not impossible without language, yet to understand the implications of complex ideas we require the help of language. Further we cannot carry on a complex process of thought without the aid of language or symbols, as in working out a difficult

mathematical problem. But even though logic requires the help of language, we must not regard it as the science of language. Conceptualists adopt the extreme position of denying that logic requires the help of language. On the other hand extreme nominalists wrongly reduce logic to the science of language. We may now show in what ways language is an aid to logic.

1. It is by means of language that we can analyse a complex notion into its constituent parts. Suppose a man is climbing a tree. The idea of a-man-climbing-a-tree is a complex idea, but the employment of language enables us to analyse it into the idea of man, the idea of the act of climbing and the idea of a tree. Similarly, substance, attribute, cause and effect etc., which are thought together, can be analysed or separated by means of language which helps the analysis of thought.

2. With the help of language, the formation of concepts becomes possible. Let us see how we form concepts. The concept 'horse' is the general idea of horse, and similarly the concept 'man' is the general idea of man. Now we form the concept 'horse' by first observing a number of horses and then comparing them to find out the common characteristics or attributes of different horses. When these common characteristics are discovered, they are abstracted or separated from other attributes of certain individuals which are peculiar to them. After abstraction we generalise these attributes and form the concept or notion 'horse.' But this notion or concept has to be named to retain it in memory and to give fixity to the concept, which otherwise remains

vague and indistinct. In this way the formation of concepts and their retention in memory becomes possible by means of language.

3. The process of thinking is simplified or shortened by means of language. When we have a number of names representing notions or general ideas we can apply them to particular cases. Besides, it is difficult to form concepts of complex matters, *e.g.* the British Constitution, the League of Nations etc. In such cases names standing for them and describing them are very useful.

4. Language is a means of communicating thought. Men could not have expressed their thoughts to one another if there had been no language. It is by means of language that the ideas of one person are communicated to others and such communication enriches the store of knowledge.

5. Language further preserves thought. We could not have access to what Plato or Aristotle or Copernicus or Newton thought if there were no language to preserve their ideas. We are rich in knowledge to-day by the knowledge of the past, and the importance of language for enshrining the wisdom of those who preceded us cannot be over-estimated. Though it is true that language as an instrument of thought is related to logic, "formal logic," as Keynes says, "is still concerned primarily with thought, and only secondarily with language as the instrument of thought."

Logic as the science of valid thinking must avoid ambi-

guity in the use of language. Words often have different shades of meaning, and ambiguity in words leads to ambiguity in thought and causes invalid thinking. Words often change their meaning by generalisation, *e.g.*, oil, which originally meant olive oil, now means any kind of oil. Similarly by specialisation, words change their meaning, *e.g.* fowl, which meant any bird, now means a particular domesticated species only. Accurate thinking depends upon the accurate use of words.

Logic must avoid the ambiguous use of words which leads to invalid thinking.

Views regarding Conception, Judgment and Inference

There are three views regarding the mind's power to form concepts or general ideas. We have already explained what a concept or notion is. But the problem here is whether the human mind can form general ideas. The three views are (1) Realism (2) Nominalism (3) Conceptualism. Plato is an advocate of realism. According to him the human mind is capable of forming concepts and corresponding to these general notions there are real essences existing in the invisible world. Thus corresponding to the notion 'virtue' which is in the mind, there is the essence or form or 'idea' of virtue in the invisible or intelligible world. Particular instances of virtue which we find in the world participate in the essence of virtue and are merely copies or shadows of that essence. Similarly corresponding to the notion or concept 'man', which is in the mind, there exists the essence of man in the real world and individual men are mere copies of that essence. Aristotle however

The three recognised views regarding Conception, viz. Realism, Nominalism, and Conceptualism, explained.

rejected this realism of Plato and pointed out that forms, 'ideas' or essences exist only in individuals and not apart from them. In modern times, those philosophers are called realists who hold that all thoughts, whether percepts or concepts, have their counterpart in the external world. (2) Nominalism. According to nominalists only names are general. It is not possible for human minds to form general notions. Whenever we think, we think of the particular and not of the general. Corresponding to a general name there may be an individual mental image or a series of such images which are concrete and not abstract. Hobbes and Berkeley are among the chief advocates of this theory. Mill, though a nominalist, holds a modified view regarding the mind's capacity to form concepts. He holds that general names fix our attention upon some common attributes of a class indicated by the name, and that in the case of intense attention these attributes may exclude all others. But he at the same time supposes that there is always present in the mind a concrete image of which the general attributes are parts. So a concept is nothing but a general image. Thus Mill's view of conception does not essentially differ from the view of his predecessors. This view regards concepts as concrete and perceptual. (3) Conceptualism. According to conceptualists (Mansel, Sigwart, Kant. Ward, Stout etc.) concepts are not sensible images but are intelligible, that is, they are mental syntheses of general attributes. Thought can be carried on by means of general ideas or concepts without the help of any image generic or other, that is, without reference to individual objects. Thinking does not necessarily imply picture thinking, that is,

thinking by means of images. General names are but symbols of general ideas, concepts or notions. It appears to us that the conceptualists are right, since we do have such notions as virtue, justice, man etc., which are not always accompanied by images but which we are justified in regarding as concepts by the fact that they are intelligible, that is, their meaning can be understood.

The three views, viz. Conceptualistic, Nominalistic and Realistic or Objectivistic, regarding judgment and inference explained.

As with regard to conception, so in regard to judgment and inference logicians hold different views. (1) According to conceptualists (Hamilton, Mansel etc.) a judgment expresses a relation between two concepts, notions or ideas, whether the relation is one of inclusion or participation. Inference according to them consists in passing from one or more judgments to a conclusion. These formal logicians define logic as the science of the pure or formal laws of thought. They mean that the matter of thought is not important for logic. According to them truth means not correspondence but consistency of thought and its freedom from self-contradiction. An idea is true, according to them, if it does not involve self-contradiction and is consistent with the totality of our thought and experience. We shall explain this view of truth more fully hereafter. This view of logic is subjective and not objective, psychological rather than realistic. (2) According to nominalists a judgment is nothing but a statement about language. Every proposition simply expresses a relation between two terms, and inference or argument consists in passing from one or more proposi-

tions to a new one, that is, from one set of relations between terms to a new set of relations. Hobbes and Whately are the chief exponents of this view. Whately however does not deny that logic is the science of thought. Nominalists reduce logic to grammar. Logic according to them is the science of names and their relations. (3) There is also a third school of logicians who are objectivists. According to them judgments express a relation between substances or between attributes or between substances and attributes. Take the proposition 'man is mortal.' According to conceptualists the judgment means that there is a relation between the concept man and the concept mortal, according to nominalists it means that there is a relation between the name man and the name mortal, while according to objectivists or realists the judgment means that there is a relation between the things or attributes signified by the term man and the things or attributes signified by the term mortal. According to objectivists inference consists in passing from one or more sets of relations between substances and attributes to a new set of relations between them. Spencer is a pronounced objectivist. He defines logic as "the science which formulates the most general laws of correlation among existences considered as objective." Objectivists reduce logic to physical science.

Mill's view regarding judgment and inference. The true nature of thought and the science of logic.

Mill, though a nominalist in his view of conception, is an objectivist in his view of judgment and inference. He partially recognises also that a proposition expresses a relation between mental phenomena. The truth of a proposition, according to him,

requires the agreement of our thought with objects. but his view is rather crude and he has been interpreted in different ways by subsequent thinkers. Without multiplying views regarding judgment and inference which may confuse the issue, we may point out that every judgment is an act of thought, and since it claims truth, it must have objective reference. Logic therefore is neither purely subjective nor purely objective. Further, since thought cannot be carried on in most cases without the help of language, logic cannot altogether ignore consideration of language and must treat of terms, propositions, arguments etc. Logic is the science of thought, and since thought is related both to objects and to language, it takes an interest, though a secondary one, in things and names.

Formal and Material Logic

The standpoint of formal logic.

A distinction is commonly drawn between formal logic and material logic as well as between formal truth and material truth. According to the formal view of logic, logic is concerned with formal consistency or formal truth, that is, it can ignore the matter of thought and studies the forms of thought alone. We have already pointed out that even physics, chemistry, geometry etc. are also in a sense formal sciences, since they require abstraction and deal with the forms of things. But since logic is more abstract than any physical science and less abstract than pure mathematics alone, it is supposed to be more formal than any other science except mathematics. Bosanquet rightly remarks that a physicist or a chemist has his laboratory and he can often appeal to sense-experiences, while a

logician has no such laboratory in which to experiment upon thought. According to formal logicians, when a judgment is free from self-contradiction it is true, and the truth of a judgment depends upon its being consistent with other judgments which are known to be true. Thus consistency of thought and freedom from self-contradiction are according to formal logicians the test of truth. According to them, a judgment need not be referred to the external reality in order to prove its validity. Thus 'No A is A,' since it involves self-contradiction, is false, while 'A is A' is a true judgment because it is free from self-contradiction and is consistent with the totality of thought. Such logicians try even to reduce inductive inference to formal treatment. We have already pointed out that every judgment is necessary and is constrained by its reference to the real world, and its truth depends as much upon consistency of thought as upon its correspondence with some aspect of reality. When we say that truth requires correspondence of thought with reality, we do not mean that ideas are mere copies of outside things. We mean that when our ideal construction of the world is accepted by the content of reality, it corresponds with reality. Logic, therefore, does not allow of purely formal treatment. The doctrine of definition, of division, of classification, of categories, of predicables etc. cannot be understood without some knowledge of reality or the universe. Formal logic is identified with deductive logic.

Its view of truth one-sided. Purely formal treatment of logic not possible.

By material logic is generally meant inductive logic. In deduction or deductive inference we pass from

general propositions to a conclusion which is less general, and here consistency is the main test of truth; *e. g.* 'All M is P, all S is M, therefore all S is P; All material bodies are extended, this stone is a material body, therefore this stone is extended. In induction or inductive inference on the other hand we pass from the facts of experience, which are particular, to a conclusion which is general; *e.g.* John is mortal, James is mortal, Joseph is mortal, etc.; therefore all men are mortal. Thus in induction we cannot omit to consider the matter of thought or the facts of the world, and our argument being based upon observation and experience of particular facts, the laws that govern it are said to be laws of material logic as opposed to formal logic. We may however point out that even inductive logic can be treated formally up to a certain point, as we shall see when we discuss the problems of induction. Thus just as formal logic cannot ignore the consideration of the matter of thought, since every judgment refers to some content of reality, so also induction cannot ignore the form of thought altogether. Therefore the distinction between formal and material logic is relative and not absolute. We may further remark that deduction and induction are not two complementary processes but are two aspects of the same process of thought. We shall find that inductive generalisations require the help of deduction for their verification and their extension to unobserved regions. Though it is true that the distinction between formal and material

The nature of material logic. The distinction between formal or deductive logic and material or inductive logic is not absolute but relative. Logic however is primarily concerned with forms of thought.

logic is not absolute, yet deductive logic can be treated of to a very large extent formally, and therefore it is identified with symbolic logic. We should also remember that though logic cannot ignore the matter of thought altogether, it is primarily or mainly concerned with the forms of thought.

Procedure and Mode of Treatment

We shall treat of the principles of deduction first as is usually done, since they are clear and simple and the knowledge of them is necessary to understand the principles of induction. Judgment being the unit of thought, the treatment of logic really begins with the discussion of judgment. Proposition being judgment expressed in language, we shall deal with its forms. But before doing that we shall treat of terms, which are the constituents of propositions, and in connection with them we shall treat of categories, predicables, the doctrines of definition, division etc. Then after treating of propositions we shall pass to the discussion of inference, which may be either immediate or mediate. Mediate inference may be either syllogistic or inductive. In this volume we shall discuss the principles of deduction only, leaving the principles of induction for another volume. We should also remember that different pieces of inference are interrelated and move towards a definite goal. At the end of the second volume therefore we shall treat of method, which is the orderly arrangement in discourse of trains of inferences. Further since thought and language are closely related we shall indifferently use the vocabularies both of conceptualists and of nominalists and shall speak

of concepts, judgments, inferences, as well as of terms, propositions and arguments, in the same sense.

The Utility of Logic

We have already shown that logic is not an art, yet its practical value is undeniable. Its function is not to lay down rules for the guidance of every particular science and teach it how to draw conclusions, nor is it concerned with helping men to argue correctly in every particular case of reasoning. The art of reasoning, that is, how to argue correctly in a particular case, does not fall within the scope of logic, just as casuistry, or the art of deciding how one ought to act in every particular instance, does not fall within the scope of ethics. In spite of all the above remarks logic is a very useful study. It not only adds to our knowledge but the abstract nature of the study of logic disciplines our mind to a high degree. Logical study is an excellent propaedeutic to the study of other sciences. Though an acute mind can detect fallacies in particular cases of reasoning, logic alone can find out why a particular piece of argument is fallacious by comparing it with the principles of valid thought. Further a systematic criticism of thought enables us to find out the valid principles of thought. When a particular science draws a false conclusion from premises, it is logic which can find out the reason why the conclusion is untenable. Further it is also largely true that right thinking leads to right action. Hamilton says that in the world there is nothing great but mind and in mind there is nothing great but reason, and a well balanced mind is the supremest possession of man. Logical study balances the human mind to a high degree.

CHAPTER II

Logic and other Sciences

General relation of logic to other sciences.

The scope of logic extends, as Johnson rightly remarks, into the domain of philosophy on the one hand and that of the sciences on the other. So no rigid distinction can be drawn between the provinces of logic and metaphysics on the one hand and the provinces of logic and other sciences on the other. Logic, it appears, is more akin to metaphysics than to the special sciences. But the spheres of logic and metaphysics should be distinguished as far as possible. Since every science must conform to the regulative principles of thought which are the study of logic, logic influences every science and therefore is rightly called the science of sciences. Though this is true, we must not suppose that the influence which logic exerts on other sciences is more than general. It does not examine the evidence or data of the different sciences but only enquires whether valid conclusions are drawn from the evidence. Since logic is the science of proof, it has the right to weigh evidence in order to determine whether conclusions have been rightly drawn from it. To find out causal laws it is necessary for every science to draw conclusions according to logical principles. Logic is called Methodology because it criticises and examines the methods employed by different sciences. So it has rightly been said that logic is practical inasmuch as it

forms an excellent propaedeutic to the study of other sciences. It has therefore been regarded by some as not merely the science of sciences but also the art of arts. But we must not forget that logic is a practical science only in a secondary sense, for its function is not to lay down rules and precepts for the guidance of other sciences. It will be clear now that logic is not only an end in itself, but provides needed guidance for the other sciences because it criticises all thought. Though logic is related to the sciences in general, it is supposed to be specially related to the science of mind (psychology), the science of being (metaphysics) and the science of language (grammar).

Relation of Logic to Psychology

"Psychology treats of psychical states and processes, their objects as such and the conditions of their occurrence." (Stout). Psychical or mental states may be either states of thinking or states of feeling or states of willing. Logic is related to psychology inasmuch as they both deal with thinking, but while psychology deals with the actual processes of reasoning, their origin and their development, logic deals with mental processes and products to find out the principles of valid thinking and to provide criteria for reasoning accurately and correctly.) Psychology regards laws of reasoning as uniformities, while logic deals with them as regulative and authoritative laws of thought, determining the formal relations in which the products of thought stand to one another. Therefore while psychology is a positive and concrete science, logic is a normative and abstract science. In other words, psychology deals

with the actual mental processes while logic deals with the ideal of reasoning. Psychology tells us how we come to believe certain things, and how one idea gives rise to another idea according to the laws of association. Logic tells us how we ought to think and believe and how our ideas should be regulated that they may be valid. Psychology deals with the origin and development of judgment and conception, logic deals with them as they ought to be, that is, with the conditions which they must fulfil in order to be valid. Since logic criticises the method of psychology, as of every other science, it may be regarded as superior to psychology. But in a sense psychology is wider than logic, because it deals with all mental phenomena, while logic deals only with the processes of reasoning or thought. There cannot be any sound logic without some knowledge of psychology. Without knowing the actual processes of thinking we cannot determine how men ought to think. Though this is true, logic is not a branch of psychology. The spheres of logic and psychology very often overlap, but logical and psychological problems should be separated as far as possible.

Logic and Metaphysics

Metaphysics or ontology investigates the nature of Being or ultimate reality. It tries to answer the question whether the things that we experience are real or only phenomenal, whether there is an ultimate reality behind the phenomenal world or the world as it appears to be. Thus metaphysics does not accept anything as real without subjecting it to searching criticism. It begins with doubt and may or may not reach a positive result; that

is, after examination of all that we know it may either say that knowledge of reality is possible or that it is not. Whatever may be the conclusions of different metaphysicians, they are all at one in criticising the presuppositions of the sciences. The sciences make such assumptions as that matter exists, that there is conservation of energy, that the things of the world are causally related, that nature is uniform, that knowledge of the laws of nature is possible, and so on. Metaphysics however cannot accept any of these assumptions as true without examination. It therefore criticises the presuppositions of every science and goes beyond it. Logic may be regarded as intermediate between other sciences and metaphysics. It criticises the conclusions of the sciences and provides criteria of validity to which every science has to conform. But it generally accepts their assumptions. It accepts certain laws of thought without criticism as axiomatic, it supposes that knowledge in the absolute sense is possible, that the object of knowledge is real and so on. Since logic enquires into the truth and falsity of thought and since thought is true only when it corresponds with some aspect of reality, logic cannot but assume that knowledge of reality is possible. But metaphysics makes no such assumptions. Its central problem is whether we can know reality or not, whether knowledge is only relative or whether absolute knowledge is possible. It enquires whether all knowledge is the knowledge of appearance or whether there may be knowledge of reality as well. Metaphysics therefore criticises the presuppositions of natural science, such as causation, the conservation of energy etc., and in the same way it examines the presuppositions of logic. Thus

though logic goes beyond all other sciences, metaphysics goes beyond logic as well. But logic cannot avoid entering into the domain of metaphysics to some extent. Logic tries to know this and that, but to have knowledge of them it requires some knowledge of the nature of the 'this' and of the 'that.' Logic however does not attempt to define the nature of reality or the ultimate being, if there be any such reality. It serves as a propaedeutic to the study of metaphysics and without some knowledge of logic the study of metaphysics is not possible. Further a metaphysician has also to reason validly to arrive at correct conclusions, and therefore metaphysics, like other sciences, is indebted to logic which studies the conditions of valid thinking.

Logic and Grammar

Since language is the instrument of thought, logic is supposed to be related to rhetoric and grammar, which are the sciences of language. Logic however cannot properly be regarded as specially related to rhetoric. which is concerned with the emotive use of language. Rhetoric is concerned with language in so far as it is intended to appeal to emotion, but logic has no reference to emotion. It is the science of reasoning and tries to find out the principles to which thought must conform in order to be true. Further logic cannot be supposed to be specially related to particular grammars. There are different grammars for different languages, but the science of logic is one, and always the same. Further, particular grammars are arts while logic is a science. Logic however may be regarded as related to

universal grammar, if there is any such thing. Mr. Johnson holds that universal grammar should be subsumed under logic because the modes in which words are combined cannot be isolated from modes of thought, and also because negation, conjunction, disjunction, implication, and alternation are modes of logic as well as of grammar. Further he says that the grammatical analysis of sentences is analogous to the logical analysis of thought. Hence he holds that grammar should be subordinated to logic. He does not however note the points of difference between logic and grammar. Logic is primarily concerned with the forms of thought and secondarily with language, but grammar is primarily concerned with the forms of language and secondarily with thought. Logic has nothing to do with non-significant words. Only significant words can form terms in a proposition. Logic takes no note of that division of words into parts of speech which is so marked a feature in the grammatical analysis of language. It is concerned only with those words which can be either the subject or the predicate of a proposition. Thus prepositions and conjunctions in their normal use can never be terms of a proposition. Grammar has long exercised a tyrannical influence over logic, but logic should be emancipated from its control as completely as possible.

CHAPTER III

The Fundamental Laws of Thought

"The laws of thought, regulative principles of thought or postulates of knowledge are those fundamental, necessary, formal and a priori mental laws in agreement with which all valid thought must be carried on." Three such laws are recognised by traditional logic, *viz*. (1) the Law of Identity, (2) the Law of Contradiction or of Non-contradiction, (3) the Law of Excluded Middle. Leibniz's Principle of Sufficient Reason is not regarded as an a priori principle of thought by logicians, for reasons which will be explained later. Besides the three fundamental laws of thought, certain other postulates of knowledge are recognised but are not held to be so important as the three main laws. These fundamental laws of thought are a priori because they are not derived from experience, and yet they are assumed in all processes of reason exercised upon the facts of the world. They are formal because they provide the main types or patterns of thought which are most general and universal and are involved in all thinking ; and also because they are not concerned with particular facts of experience and cannot by themselves ascertain the properties of them. They are necessary because we cannot think of them as reversed, nor can we knowingly violate them. We commit fallacies no doubt, but we do so out of ignorance, not knowingly. The three fundamental laws are postulates of knowledge because without them knowledge cannot be

systematised. They resemble scientific uniformities, but differ from the latter inasmuch as they are not derived from particular experiences and are self-evident, that is, their truth is not to be proved by other judgments. But though these laws are self-evident, they do not provide any criterion or standard of how men ought to think. They only describe thought as it is. Keynes rightly remarks that the function of these laws is negative rather than positive. They give us principles by conforming to which we may avoid fallacies, but they do not lead to any positive result. Hamilton and others are wrong in trying to deduce all processes of valid thinking from the three laws of thought. We may now explain these laws.

The Law of Identity

The simplest statement of the principle of identity is 'A is A'. Leibniz states it as 'Everything is what it is' and Jevons as 'Whatever is, is.' The conception of identity is meaningless if it does not imply diversity as well. 'A is A,' to be significant, must imply diversity. In this case there is diversity because the two letters A occupy different points in space and yet they are thought of as the same. Thus when we say that a thing is identical with itself we mean that it remains the same amidst diversity of circumstances. We do not mean by the principle of identity that two things are the same without any attendant difference, which is nonsense; we only mean by it 'identity of one thing amid diversity in other things.' Sigwart says that the law implies, that "Truth is something fixed and invariable." Bradley points out that

the law of identity signifies that truth is at all times true; once true always true, once false always false. Thus the principle of identity tells us that if a judgment or term is repeated several times its truth is not affected. {Mill as a nominalist holds that the principle means that whatever is true in one form of words is true in every other form of words which conveys the same meaning, and that the same term should be used in the same sense throughout a discourse.}

The Principle of Contradiction

The principle of contradiction or of non-contradiction may be best stated as 'A cannot both be B and not be B'. Jevons states it as 'Nothing can both be and not be.' It is also stated as 'A cannot be not-A.' The principle means that the two contradictory judgments 'A is B' and 'A is not B' cannot both be true. The same subject cannot accept two incompatible attributes, e.g. a thing cannot be green and not be green. The truth of this principle is not affected if different parts of the same surface are green and not green, because here the attributes are referred to different points in space. The same portion, say, of the wall of a house cannot both be green and not be green. Again the truth of the principle is not affected if the subject accepts incompatible attributes at different points of time. Thus a piece of iron may be hot at one time and not be hot at another time. But since here reference is to different points of time, we have really two propositions, viz. that at such and such a time a piece of iron is hot and that at some other time it is cold. The principle is applicable when incompatible attributes are referred to the

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same subject at the same time, that is, two contradictory propositions cannot both be true at the same time. The principles of identity and of contradiction form the basis of all immediate inference and of some cases of mediate inference. Mill's interpretation of the law is that "the affirmation of any assertion and the denial of its contradictory are logical equivalents which it is allowable and indispensable to make use of as mutually convertible."

The Law of Excluded Middle

This principle may be well stated as 'A either is, or is not, B.' It has also been stated as 'A is either B or not-B'. The principle means that one of two contradictory propositions must be true. The principle of contradiction points out that two contradictory propositions cannot both be true. The principle of excluded middle supplements it by stating that one of them must be true. Thus a piece of iron must either be hot or not be hot; a man must either be honest or not be honest. One of the two propositions 'A is B' and 'A is not B' must be true. We need not like Sigwart deduce the principle of excluded middle from the principle of contradiction and that of double negation taken together. His principle of negation is that if we deny a negative proposition we get its contradictory affirmative proposition. Thus if the truth of 'A is not B' is denied or negated, then the truth of 'A is B' is to be affirmed. Jevons tries to combine the principle of contradiction and the principle of excluded middle by his 'principle of duality.' If two alternatives are exclusive and exhaustive, then they cannot both be true of the same subject at the same time,

and one of them must be true of the subject. But it is better to keep the two laws independent without combining them into one. Just as in the principle of contradiction the reference to the same subject and to the same point of time is necessary, so also in the principle of excluded middle the reference must be to the same subject and to the same point of time. The truth of the principle of excluded middle is denied by some. But these persons confuse contradiction with contrariety. It is pointed out that a flower need not be either white or black but may be red. A thing need not be either greater or less than another thing but may be equal to it. Similarly Mill says that between true and false propositions there may be unmeaning propositions. But in the examples given, the alternatives predicated of the same subject are not contradictories. While contraries mark the utmost divergence between attributes, contradictories negate each other. Thus it is undeniable that a flower must either be green or not be green, a thing must either be greater or not be greater than another thing. In reply to Mill we may point out that an unmeaning proposition is no proposition at all. Thus the principle of excluded middle is as true as the other principles. Mill states the principle as, "It is allowable to substitute for the denial of either of two contradictory propositions the assertion of the other." We shall find that some instances of both immediate and mediate inference depend upon this principle.

Mr. Joseph regards these laws of thought as ontological laws. So he says that "The connection between ques-

tions about our thinking, and whatever we must think things to be, is excellently shown in the so-called laws of thought." These laws show that logic and metaphysics are closely related. According to Kant change presupposes permanence, and this is the essence of the law of identity. The law of contradiction says that a thing cannot have opposite characters. The law of excluded middle states that it must have one of these characters. Since there cannot be any determination without negation, the principle of excluded middle is also necessary.

The meaning of the three laws explained by comparison. These laws show that there is a close connection between logic and metaphysics because they are as much laws of thought as of things.

Joseph sums up the implication of the three laws in the following words :—"If we think about anything then (1) we must think that it is what it is ; (2) we cannot think that it at once has a character and has it not ; (3) we must think that it either has it or has it not.") From the standpoint of language the three principles may be summed up in the following words of Welton and Monahan : "Whenever we use a term we must be understood to use it unambiguously both (1) positively and (2) negatively, and (3) it must either be given or denied to everything whatever. That is, the use of a term asserts all the attributes it implies, and denies all others which are incompatible with them ; and everything must either possess all those attributes or be without some, or all, of them."

All these three laws are equally fundamental. We should not suppose that any of them is more important

The three laws are equally important and necessary for knowledge.

than the others. Some logicians regard the law of identity as more fundamental than the other two laws, while others suppose that the principle of contradiction is more important than the other two. But all three are equally necessary for our knowledge of the facts of the world. The three laws together imply that "I affirm what I affirm, and deny what I deny; if I make any affirmation, I thereby deny its contradictory; if I make any denial, I thereby affirm its contradictory." (Keynes).

The Principle of Sufficient Reason

Leibniz holds that the Principle of Sufficient Reason is as fundamental as the principles of identity, of contradiction and of excluded middle. He states the principle thus: "Whatever exists or is true must have a sufficient reason why the thing or proposition should be as it is and not otherwise." The principle may be symbolically expressed as $A+B=C$; *e.g.*, gunpowder and fire=explosion. It is a postulate of thought and implies that things in the world are causally related. The law may mean either that there must be some logical ground to explain why a proposition is true, or that every event has a cause. It may be regarded as the foundation of syllogistic as well as of inductive reasoning. Since Leibniz regards the principle of sufficient reason as implying mainly that the things of the world are causally related, it is therefore not sufficiently formal and cannot be regarded as a fundamental law of thought like the three principles of Aristotle.

We shall discuss in its proper place Aristotle's 'dictum de omni et nullo,' which is a postulate of syllogistic reasoning. The principle of causation, which is supposed to be the formal ground of induction, will be treated of in connection with inductive inference.

Some other postulates of knowledge recognised.

Hamilton gives us what he calls the Postulate of Logic. He says: "Before dealing with a judgment or reasoning expressed in language, the import of its terms should be fully understood, in other words, logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought." Since Hamilton allows the same meaning to be expressed by different words, his postulate is almost the same as Mill's reading of the law of identity. There may be principles other than the fundamental laws of thought upon which valid reasoning may depend. There are some such mathematical principles, *e.g.*, the argumentum a fortiori. 'If A is greater than B and B is greater than C then A is greater than C'; 'things equal to the same thing are equal to one another.' But we need not enumerate any other postulates of knowledge enunciated by logicians.

BOOK I

TERMS; DEFINITION; DIVISION AND CLASSIFICATION

CHAPTER I

TERMS

Introductory Remarks

Knowledge has its object.

Knowledge really begins with judging and every **judgment** has reference to reality, that is it is concerned with matter of fact. Though sensations provide the raw materials of knowledge yet it is undeniable that knowledge begins, only when we judge about them. All our judgments are based upon experience, and experience is experience of reality. Therefore it can be said that knowledge necessarily has reference to reality. Mere subjective attitude cannot create knowledge. So Bosanquet remarks that "knowledge is the mental construction of reality."

Judgment, Proposition and Sentence

It is generally held that a **proposition** is the verbal expression of a judgment, but this view appears to be erroneous, since a mere verbal expression can neither be true nor false. According to Bosanquet, "Judgment claims to be true, *i.e.* presupposes the distinction between truth and falsity." He also holds that "Judgment expressed in words is a proposition." But this view is not correct, because the mere expression of a judgment in language is

a sentence and not a proposition, and every proposition must be either true or false. Mr. Russell defines a proposition as "anything which is either true or false." Johnson also holds that "a proposition is that of which truth and falsity can be significantly predicated." If a proposition is merely the verbal expression of a judgment, we cannot characterise it as either true or false. It is the function of judgment to say that a proposition is either true or false. Thus according to Johnson proposition is not self-subsistent but only a factor in the concrete act of judgment. **Judgment** according to him is, therefore, subjective or **epistemic**, that is, it provides the psychical aspect of knowledge, while **proposition** is objective or **constitutive** in the sense that it provides the objective aspect of knowledge. Much confusion in logic is due to the failure of logicians to distinguish between judgment and proposition, and we intend to discuss the matter more fully when we treat of propositions.

The parts of a proposition: Subject, Predicate and Copula.

A proposition is a statement about reality. An analysis of a proposition gives us **three parts,** viz. **Subject, Predicate,** and **Copula.** Something of which affirmation is made is called the subject, something that is affirmed or denied of the subject is called the predicate, while the verb 'to be' (whether alone or accompanied by 'not'), by which the assertion is made, is called the copula. Let us take the proposition 'Socrates is wise.' Here 'Socrates' is the subject, 'wise' is the predicate and 'is' the copula. Again in the proposition, 'Man is not a four-footed animal,' 'man' is the subject, 'four-footed animal'

the predicate and 'is not' the copula. We shall hereafter find that the copula is not a third term in a proposition. Every fully developed proposition has only **two terms, viz.** the **subject** term and the **predicate** term.

Logical implication of terms.

Jevons holds that the doctrine of terms cannot properly be regarded as a logical doctrine. But this view cannot be accepted, since the meaning of a proposition cannot be understood without grasping the significance of terms. But terms are the constituents of a proposition, and the implication of the logic of terms can be understood only when we remember that terms have no existence in logic except as parts of a proposition.

Terms and Names.

Some logicians use 'terms' and 'names' in the same sense. A **name** is a symbol of some substance or attribute, but all names are not terms. A name in order to be a logical term must either be the subject or the predicate of a proposition. Therefore all names are not terms though they may function as terms: that is, isolated names which are not parts of any proposition are not terms.

Single-worded and many-worded terms.

A term may consist of a single word or of a combination of words. 'The Secretary of State for India is an Englishman' is a proposition in which the subject term, 'the Secretary of State for India,' and the predicate term, 'an Englishman,' are both many-worded terms. But the terms 'happiness' and 'desired' in the proposition 'happiness is desired' are both single-worded terms.

Categorematic and syncategorematic words.

A **categorematic** word is one which by itself can serve as a term, while a **syncategorematic** word is one which cannot itself function as a term but can become a term only when joined with a categorematic word. Every word must be either categorematic or syncategorematic. Logic ignores the grammatical division of words into parts of speech. It is plain that the nominative and possessive cases of nouns and pronouns are categorematic words, and so also are adjectives and participles, while the objective cases of nouns and pronouns are syncategorematic words. Adverbs, articles, prepositions, conjunctions and interjections are syncategorematic words because normally they cannot by themselves serve as terms. Adjectives can always be used as predicates, but not as subjects except by an ellipsis, as in 'the poor are miserable.'

Thought and Language.

Mill is wrong in supposing that inference is impossible without the help of language, for in chess playing we often infer without the help of symbols. But since thought and language are intimately connected and in thinking we very often require the help of language, we are justified in treating of propositions and terms which have reference to language.

Classification of terms.

Terms are classified by logicians as—

1. Singular and General ;
2. Collective and Distributive ;
3. Concrete and Abstract ;
4. Positive and Negative (Privative) ;
5. Absolute and Relative ;

6. Particular and Universal ;
7. Univocal, Equivocal and Analagous ;
8. Connotative and Non-connotative.

In this chapter we shall discuss the first seven divisions. We shall devote a separate chapter to the last division, viz. connotative and non-connotative terms. Of these divisions the first, third, sixth and eighth are important for logic. Though each division is exhaustive, the distinctions are not independent of one another. Though a term cannot be both singular and general, it can be general, concrete and connotative at the same time.

Singular and General Terms

Terms may be either **Singular** or **General.** According to Keynes, "A singular or individual name is a name which is understood in the particular circumstances in which it is employed to denote some one determinate unit only," while "a general name is a name which is actually or potentially predicable in the same sense of each of an indefinite number of units."

Definition of Singular and General terms.

A singular term which refers to one thing only may be either a **proper name** or a **designation,** that is, a uniquely descriptive name, as Welton calls it. A proper name is a sign or a mark which is predicated of an individual thing without signifying attributes which may be possessed by that thing. It is a mark of identification only. The term 'London' is the name of a place and it does not from the logical point of view signify any attribute which the place may possess.

Singular terms either proper names or uniquely descriptive names. Nature of a proper name.

such as its situation, its importance, etc. A proper name may suggest certain qualities by association, *e.g.*, the name 'John' may suggest the appearance of John to one who knows him. But that is a psychological fact which should be distinguished from the logical implication of a name. 'John' as a proper name is a mark to identify a person and does not refer to any quality which that person may have.

The nature of a proper name is not impaired even if many objects bear the same name.

It is no doubt true that different places or persons or things may have the same name, but the same proper name is not given to a number of objects because they possess some common attribute. In each use of the same name 'Jane,' whether it refers to a woman or a dog or a ship, it has a unique reference and points to one object only.

Russell's view of proper names.

Mr. Russell holds that the terms 'that,' 'this,' 'here' and 'now' are the only proper names, since such terms refer merely to a particular thing or space or time without reference to any quality. But this is not an absolutely correct view; and such names as 'Calcutta', 'the Ganges', 'Motilal', etc. can properly be called proper names from the logical point of view, since each of them uniquely refers to some object without referring to any quality.

Attributive names can function as proper names.

In 'honesty is a virtue', 'honesty' may be regarded as a proper name because it refers to one definite attribute. Again 'the red of this flower is different from the red of that flower' is a proposition in which the term 'the red' can from the logical standpoint be regarded in both

its uses as a proper name, since it uniquely refers to only one example of a certain quality.

Significant singular names, also called **designations** or **uniquely descriptive** names, should be distinguished from proper names. A uniquely descriptive name refers to one object only, not, like a proper name, directly, but indirectly through its meaning. Thus 'the present leader of the Indian congress' refers to one man only, because he possesses certain attributes. Here the reference is to an object through its meaning. Significant singular names are given to things because it is not always convenient to give proper names to all objects. Some uniquely descriptive names possess a time reference. In the proposition 'This man is young,' 'this man' refers to a particular person at a particular point of time, since he will not remain young all through his life. But there are other significant singular names which involve no such time reference, *e.g.* 'the first Stuart king of England,' 'the inventor of the mariner's compass.' Often for the sake of convenience we use significant singular names even when reference is made to objects having proper names, *e.g.* we say 'I am going to the river,' meaning the Ganges.

A significant singular name refers to one thing only indirectly through its meaning. The characteristics of uniquely descriptive names.

We have already pointed out that a **general name** is applicable in the same sense to each of a number of similar objects, whether such objects be actual or potential. Such names are also called **class names,** since the reason for giving a general name to a number of objects

Nature of a General term.

in the same sense is that they have certain characteristics in common. From the subjective point of view such terms stand for concepts, and from the objective point of view every general term may be affirmed of a number of similar things. Thus man, dog, house, king etc. are general terms, and they are predicable of a number of objects, *e.g.*, 'John is a man,' 'James is a man,' etc.

General names and uniquely descriptive names, similar yet different.

A general term resembles a uniquely descriptive term inasmuch as they both refer to objects indirectly by signifying some attribute or attributes. But whereas a general term can be affirmed in the same sense of a number of objects, a designation or a significant singular term can be affirmed in the same sense of one thing only.

A general name may be transformed into a significant singular name by means of an individualising prefix.

A general name can be transformed into a uniquely descriptive name by means of some individualising prefix. 'Man' is a general term but 'this man' is a singular term. Here the addition of the individualising prefix 'this' has transformed the general term 'man' into a uniquely descriptive term.

A name may be general if it refers to possible or imaginary objects having a common meaning.

According to Welton such terms as 'Conqueror of England' or 'Emperor of Switzerland' are general names. The term 'Conqueror of England' is general, though applicable to William I only, because it is potentially applicable to others, that is, it is conceivable that there might be other conquerors of England as well. The term 'Emperor

of Switzerland' is also general, though Switzerland had no emperor, because it is conceivable that a number of persons might or may be emperors of Switzerland.

"In class-terms the unity which in individual names is one of application is restricted to content or meaning, in application it is overshadowed by the idea of plurality" (Welton). This means that a general term is applicable to a number of objects because they have a common content and such a content gives unity to a general term, while a singular term, since it applies to one thing only, provides us with an example of unity in application.

The nature of unity of a general and of an individual name.

Collective and Distributive Terms

Keynes defines a **collective** name as "one which is applied to a group of similar things regarded as constituting a single whole," *e.g.* army, navy, the Himalayas, alphabet, library, the House of Commons, etc.

Collective name defined.

The constituents of a group which can have a collective name must bear a resemblance to each other. A fortuitous concourse of heterogeneous objects cannot have a collective name. It is wrong to contrast collective names with general names, to regard them as a sub-class of singular names, to postulate a three-fold division into singular, collective and general, since a collective name may be either singular or general. A collective term is not a distinction within the class of terms singular and general.

Its relation to general and singular names.

There are collective names which are similar to proper names, such as the Himalayas, the Alps, the Pyrenees, etc. There are other collective terms which are of the same nature as uniquely descriptive names or significant singular names, *e.g.* the British navy, the Sanskrit alphabet, Hindu society, etc. 'The 40th Regiment of foot' is a singular collective name, but the term 'regiment' is a general collective name, because there are numerous regiments and the term 'regiment' can be applied to each of them.

Collective terms may be either proper names or significant singular names or general names.

Such terms as water, gold, silver etc. are **substantial** terms and are collective in a peculiar sense: they refer to homogeneity in spite of indefinite divisibility. According to Keynes such collective terms as water, gold, etc. are not singular but general, since water or gold admits of variety.

Substantial terms collective and general.

Though we cannot assert incompatibility between a collective and a general term, yet there is a real distinction between the **collective** and **distributive** use of names. "When we use a term collectively our assertion will only apply to the group as a whole; when we use it distributively we assert something about each member of the group individually." (Welton and Monahan). When we say that 'the British government has decided to enter into an economic agreement with the United States' we use the term 'British government' collectively. But when we say that 'the British

The distinction between collective and distributive use of names explained by examples.

government are trying to settle a dispute between the employers and the employed' we refer to the members constituting the government distributively, and here the term 'the British government' has its distributive use. Again when we say that 'all the books on the shelf weigh 10 seers,' we use the term 'all the books' collectively, but we use the term distributively when we say that 'all the books on the shelf are text-books of logic.' Again, 'all the angles of a triangle are equal to two right angles' is a proposition exemplifying the collective use of the term 'all the angles,' while the same term is used distributively in the proposition 'all the angles of a triangle are less than two right angles.'

Concrete and Abstract Terms

Definition of abstract and concrete names given by Logicians.

Mill states that "A **concrete** name is a name which stands for a thing; an **abstract** name is a name which stands for an attribute of a thing." But if this definition of Mill's is accepted, then his view that such terms as 'white,' 'red', etc., that is, adjectives, are concrete becomes untenable, since such terms are names of attributes.

Explanation of the distinction between concrete and abstract.

Welton's definition of concrete and abstract terms is better, since it avoids this confusion. According to him, "A concrete term is the name of an object," while "an abstract term is the name of an attribute considered by itself." If this definition is accepted then all adjectives, which are names of attributes, no doubt become concrete, since when we think of white,

red, etc. we think of them as attributes of certain objects. But when an attribute is considered by itself without reference to something of which it is the attribute, the name of such an attribute may be called an abstract name, *e.g.* whiteness, virtue, squareness, triangularity etc. Attributes always belong to objects but they may be thought of either in relation to them or by themselves. In the former case they are concrete, in the latter they are abstract. From the metaphysical point of view substance is that which has attributes, while an attribute is that which belongs to some substance.

It is possible for men to think of attributes as residing in some subject or to think of them without reference to any subject. The latter process of thought is called abstraction. When we say 'Milk is white' we think of the attribute 'white' as a characteristic of milk, and therefore the term 'white' is concrete. But when we say 'whiteness is a pleasant colour,' we think of whiteness by itself and therefore 'whiteness' is abstract. But we should consider another matter in this connection. There may be attributes of attributes, that is, an attribute may be a subject which is qualified by attributes. An attribute which is thus qualified by attributes is regarded by some as concrete; *e.g.* in 'virtue is desirable,' 'whiteness is pleasant to look at,' 'patience is praiseworthy,' 'virtue,' 'whiteness,' and 'patience' are said to have been used concretely. So it may be said that some terms are always concrete, *e.g.* man, Philip, Calcutta, the best Indian; while others may be either concrete or abstract according to use, *e.g.* whiteness, virtue, colour, patience etc. To meet this necessity

Keynes gives us a modified definition of concrete and abstract terms.

Modified definition of abstract & concrete names by Keynes. The real distinction is not between abstract and concrete terms but between abstract and concrete use of terms.

According to him, "An abstract name is the name of anything which can be regarded as an attribute of something else (whether it is or is not itself a subject of attributes), while a concrete name is the name of that which cannot be regarded as an attribute of something else." From what we have said above it will be clear that the distinction between concrete and abstract terms is not absolute. But if we want to make them exclusive, we should distinguish between the concrete and the abstract *use* of terms.

Different kinds of abstract names.

An abstract term may be the name of an **attribute,** *e.g.* colour, virtue, whiteness etc. It may be the name of a **universal**, which according to Joseph should not be regarded as an attributive name, *e.g.* humanity, triangularity, squareness etc. An abstract term again may be the name of some **relation**, *e.g.* paternity, equality, causality, reciprocity, identity etc.

Abstract and concrete terms go in pairs.

It is held by logicians that concrete and abstract terms generally go in pairs; *e.g.* beautiful—beauty, powerful—power, white—whiteness, strong—strength, man—humanity, triangle—triangularity, father—paternity, etc. In all these cases the first term of each pair is concrete, while the second is abstract.

We may here observe that proper names, though they

Proper names concrete.

do not indicate the possession of any attribute, are concrete terms. But the objects designated by proper names can be thought of as possessing attributes. Objects which have proper names have potential attributes.

Grammatical, psychological and metaphysical distinction between concrete and abstract terms.

Welton points out that from the grammatical point of view adjectives are concrete terms, while abstract nouns are abstract terms, e.g. good—goodness. From the psychological point of view abstraction involves a process of thought by which attributes are separated from the objects to which they belong. Attributes signified by abstract terms, therefore, may be thought of by themselves without reference to objects, while a concrete term implies objects thought of as possessing attributes. Every class name such as man, dog, etc. stands for a concept which is formed by abstraction. This however is not a reason for regarding every class name as abstract, as Locke did. Psychological process should be distinguished from logical implication. From the metaphysical point of view concrete names are given to substances, or objects having attributes, or to wholes having parts, while an abstract name is given to attributes belonging to substances, or to parts which are the parts of a whole, when such attributes or parts are considered by themselves.

Discussion as to whether abstract names are singular or general.

Substantial names may be either general, e.g. man, house, etc., or singular, e.g. the House of Commons, John, James, etc. But there is a good deal of controversy as to whether abstract names are singular or

general. According to Mill such abstract terms as admit of variety are general; e.g. colour, because there may be different colours; whiteness, since there are different shades of whiteness; virtue, because there are different kinds of virtue; while those abstract terms which do not admit of variety are singular, e.g. milk-whiteness, visibility, squareness, equality etc. But Welton rightly points out that whiteness and milk-whiteness should both be regarded as singular, since both whiteness and milk-whiteness refer to one attribute only. Some logicians have therefore regarded all abstract names as singular. At any rate those logicians are really in the wrong who hold that all abstract terms are general, on the ground that every attribute may be possessed by a number of objects; for we give abstract names to attributes not because they belong to objects but because they are thought of by themselves. But it is undeniable that when concretely used abstract names may be general; e.g. in 'some virtues are more praised than others,' 'all yellows are pleasant looking', the terms 'virtue' and 'yellow' are used in a general sense. So in deciding whether an abstract term should be regarded as general or as singular we must be guided by its use in the particular instance: no merely formal rule can be framed. From the above discussion it is clear that the distinction between abstract and concrete terms is based upon grammatical and psychological grounds rather than upon logical considerations.

Positive and Negative Terms

According to Keynes, "A **positive** name implies the presence, in the things called by the name, of a

Definition of positive and negative terms.

certain special attribute or set of attributes, while a **negative** name implies the absence of one or other of certain specified attributes." Welton, to show the distinction between positive and negative terms, says, "A positive term implies the presence of an attribute, or group of attributes, and a negative term simply implies the absence of the attributes connoted by the corresponding positive term, but implies the presence of no attributes whatever." The distinction between positive and negative terms may be brought out by contrasting pairs of contradictory terms. Thus if A is positive, not-A is negative; white is positive, not-white negative; good positive, not-good negative; man positive, not-man negative.

Examples.

"All terms whatsoever which imply attributes which cannot co-exist in the same subject are **incompatible.**" This incompatibility can be expressed by

Various kinds of incompatible terms.

either **contradictory**, **contrary**, or **repugnant** terms. We shall explain the different kinds of incompatibility by means of examples.

The distinction between a positive and its corresponding negative term is wrongly regarded by some as purely formal.

The meaning of contradictory terms as exemplifying the distinction between positive and negative terms.

This distinction, as we have remarked, is brought out by contradictory terms—A and not-A, white and not-white, etc. (Two terms are **contradictory** when they are exclusive and together cover a particular universe of discourse.) Thus 'white' and 'not-white'

are exclusive and yet exhaustive, covering the whole universe of colour. Similarly 'man' and 'not-man' are exclusive and they together exhaust the universe of living things.

Some further remarks upon contradiction.

There is a good deal of controversy as to the nature of a negative term. We may here remark that two incompatible terms cannot be predicated of the same subject. Two contradictory terms can neither both be true nor both be false of the same subject at the same time. A man cannot be both an Indian and a non-Indian at the same time, and he must either be an Indian or a non-Indian. Now, what is the meaning of a negative term, say, not-A? According to Mill, if a positive term is significant, the corresponding negative term must also be significant, because the non-possession of an attribute is itself an attribute. Thus if A is significant, not-A is also significant. According to Aristotle negative terms are logical figments, they are names indeterminate. If not-A means nothing, then such a term cannot be predicated of any subject and is therefore useless. Welton also holds that a negative term simply signifies the absence of the quality implied by its corresponding positive term. From the formal point of view a negative term is indefinite and infinite, because not-A does not signify any attribute in particular and may signify an infinite number of attributes not signified by A; *e.g.* if A implies 'red,' then not-A may imply not only white, blue, but also virtue, square, triangle, man, dog etc. But from the material point of view a negative term cannot be regarded as

Formally negative terms indefinite and infinite.

either indefinite or infinite. 'Not-white' means any colour other than white and also refers to a limited universe of discourse, *viz.* colour. 'Not-white' does not mean virtue, justice, square, and everything else which does not possess the attribute 'white'. It means a colour which is other than white. So Bradley says that every negation has a positive background; such a positive meaning however is not explicit in a negation though it is referred to by it. Welton also points out that a negation is neither indefinite nor infinite. If 'not-heavy' may mean yellow, then it is not the contradictory of 'heavy', since gold is heavy yet yellow. Keynes also holds that since negation always implies a particular universe of discourse, it cannot be meaningless, because it has a definite denotation, that is, can be interpreted as indicating a definite number of objects. Thus 'not-white' denotes or indicates all the colours other than white. We cannot therefore agree with Welton that a negative term implies the absence of some attribute and not the presence of any attribute whatever. We should rather simply say, with Keynes, that a negative term implies the absence of some specified attribute.

But from the material point of view negative terms are significant and every negation implies a positive background.

The fact is that to understand clearly the meaning of a negative term we must regard it as the predicate of a judgment. When we say that 'this man is not tall' we certainly do not mean that he has no quality at all. What we do mean is that he has a certain height which is incompatible with that which

The meaning of a negative term can be understood if we regard it as the predicate of a judgment.

we call tall. In passing we may remark that Keynes holds that even from the material point of view we may regard a negative term as indefinite and infinite. Thus 'non-moral' can be predicated of man, dog, stone, heavenly bodies etc. But this is an exception and such a term is not restricted to any universe of discourse. Our general remark that from the material point of view a negative term is significant holds good none the less. Joseph is however right in holding that since 'this flower is not-white' means the same thing as 'this flower is not white,' it is useless from the logical point of view to use negative terms.

Keynes' view regarding negative terms as infinite and indefinite.

Negative terms logically useless.

Language has devised terms which have the appearance of negative terms but which cannot properly be regarded as negative. Such distinctions as happy—unhappy, pleasant—unpleasant, convenient—inconvenient, holy—unholy, sincere—insincere, sensible—senseless, fortune—misfortune, etc., are necessary from the practical point of view. None of these pairs, however, is an example of contradictory terms, because they are not exhaustive. Between 'pleasant' and 'unpleasant' there may be an indifferent state of feeling. Again 'unhappy' does not merely mean absence of happiness but it also means the presence of pain. Similarly 'unpleasant' has a positive meaning. So these terms may be regarded as positive, though negative in appearance. Mill remarks that 'idle' is a negative term because it means 'not active', and similarly 'sober' is a negative term meaning 'not drunken'. But we may

Some terms negative in appearance but positive in meaning.

point out that both 'idle' and 'sober' have a positive meaning, each implying some particular state. Mill is wrong in regarding such terms as negative.

Some remarks regarding formal and material contradiction

In the case of formal contradiction, if we know one term, we know the other as well. If we know A, we know its contradictory, not-A. If we know green we know its contradictory, not-green. But in the case of material contradiction, which is not contradiction proper, this is not so. If a thing is not greater than another thing, it need not be smaller than the latter but may be equal to it. Similarly if a man is not pleasant he need not be unpleasant. Such terms as right—left, male—female, high—low, good—bad, happy—unhappy are examples of material contradiction. The formal contradictory of a positive term is obtained by prefixing 'not' or 'non' to it. The contradictory of happy is not-happy, of A, not-A.

Contrary terms.

"Two terms are usually spoken of as contrary to one another when they denote things which can be regarded as standing at opposite ends of some definite scale in the universe to which reference is made." (Keynes). Thus contrary terms show extreme divergence, *e.g.* first—last, male—female, black—white, right—left, wise—foolish etc. Contrary terms though exclusive are not exhaustive. A thing may be neither black nor white, but green. Two contrary terms, being incompatible, cannot both be true of the same subject, but neither need be true of it. A man cannot be both to my right and to my left, but he

may be neither to my right nor to my left. But of two *contradictory* terms, one must be true of a subject.

A distinction is drawn between contrary terms and terms which are simply **repugnant.** Two contrary terms, according to strict definition, must stand at opposite ends of a scale. But there are terms which are incompatible, yet do not stand at opposite poles of a particular universe of discourse. Thus black and white are opposite, while green and red, though incompatible, do not stand at opposite ends of a scale. Thus green, red, white are repugnant terms. Similarly 'not-gold' may imply silver, copper, iron etc., which are neither contrary nor contradictory but are repugnant. Logicians generally ignore the distinction between contrary and repugnant terms, since both alike are exclusive and yet not exhaustive. Following the general opinion, we may regard both repugnant and contrary terms as contrary. Thus we may regard both white and black, and green and red, as pairs of contrary terms.

Repugnant terms need not be logically distinguished from contrary terms.

A **privative** term implies the absence of an attribute in a subject capable of possessing it. A privative name, according to Mill, is equivalent in its signification to a positive and a negative name taken together. Such terms as blind, deaf, dumb, lame are privative terms. We call a person deaf because he lacks the capacity of hearing which we expect a human being to possess. We do not call a stone or a tree deaf. A man who is devoid of sight is called blind, and this term is privative because he might conceivably have had sight. A privative term means two things, *viz.* the absence of a certain attribute and

Privative terms.

the presence of others from which the presence of the former might naturally be expected. Such terms as unhappy, ignorant, cruel, unkind may also be regarded as privative. We call a man unkind, since, though kindness is a quality he might possess, he lacks it when he is unkind. From the logical point of view privative terms have no importance because they rest upon the psychological fact of expectation.

Absolute and Relative Terms

Terms are further classified as Relative and Absolute or non-relative. Mill prefers to avoid the term 'absolute', since it has a good deal of metaphysical significance and uncertainty. Such terms as father, son, like, unlike, equal, unequal, ruler, subject, husband, wife, partner, friend, etc. are **relative** terms, while man, dog, house, tree, etc. are **absolute** terms. According to Keynes, "A name is said to be relative, when, over and above the object that it denotes, it implies in its signification another object, to which in explaining its meaning reference must be made." So it may be said that relative terms go in pairs. 'Father' implies 'son', 'husband' 'wife'; similarly like implies like, equal equal, unequal unequal, friend friend, and so on. Mill says, "Every relative name which is predicated of an object, supposes another object (or objects), of which we may predicate either that same name or another relative name which is said to be the **correlative** of the former." Thus son is the correlative of father and father of son. Each of the two related terms is called the correlative of the other.

Definition of relative and absolute terms, with examples.

There is a common ground or basis upon which correlative terms rest. According to Mill, "All that appears necessary to account for the existence of relative names, is, that whenever there is a fact in which two individuals are concerned, an attribute grounded on that fact may be ascribed to either of these individuals."

Correlative terms have a common ground.

This attribute or ground of relation of two correlative terms is called by the Schoolmen the *fundamentum relationis*. The ground of relation between two partners is the fact of partnership, between two friends friendship, between husband and wife the fact of the marriage tie. Thus, according to Welton, "Paternity and sonship are not two different facts but the same fact viewed from two different sides and connoted both by parent and by son". Though 'friend' and 'like' are relative terms, 'friendship' and 'likeness' are not, since they are the grounds of relation. According to Mill a relation such as paternity, likeness, friendship is an attribute like any other attribute, say, whiteness, goodness, redness etc. "To predicate of A that he is the father of B, and of B that he is the son of A, is to assert one and the same fact in different words. Father connotes the fact, regarded as constituting an attribute of A; son connotes the same fact, as constituting an attribute of B."

Relation according to Mill is an attribute.

Relative terms are general, because every relative term admits of a variety of instances. Thus many persons may be regarded as fathers.

Relative terms general.

Jevons argues that since water is related to its constituent elements, gas to coal, a tree to the soil, every term is in a sense relative. We cannot understand the meaning of a term except by distinguishing it from another term. The psychological law of relativity also tells us that we can understand a particular state of mind only when we relate it to other mental states. Moreover, every term implies its contradictory—A implies not-A, white not-white. All notions and all things are thus relative. Nevertheless Jevons admits that when we call a term relative, we mean a special kind of relation which it bears to another term. Keynes, in replying to Jevons, holds that when we regard a name as relative we do not mean that the object it signifies cannot be thought of without some other object, or that it depends upon other objects, but we mean that the signification of a relative term cannot be understood without reference to something else which is called by a correlative name, as in the case of father and son. Thus we can agree with Joseph that only the totality of existence is absolute from the metaphysical point of view, but from the logical standpoint we can regard such terms as man, house etc. as absolute, since we can understand the meaning of the term 'man' without direct reference to the meaning of any other particular term.

Examination of Jevons' view that all terms are relative.

Particular and Universal Terms

Mr. Russell and some other logicians have drawn a distinction between **particular** and **universal** terms. From the

logical point of view this distinction is important because it is concerned with the form of propositions. According to Welton and Monahan, "For logic a particular term is a term which is not the relating element in a proposition, while any term which is a relating element in a proposition is a universal." This will be clear if we consider some examples. In the proposition 'Socrates is honest' the relating element is 'honest' and therefore it is universal, and 'Socrates' is a particular term because it is not the relating element in the proposition. This is an example of **monadic** or **one-termed relation,** *i.e.*, an attribute, since the relating element in the proposition is related to one term only. In the proposition 'this book is on the table', the term 'on' is the relating element and is universal, while the terms 'this book' and 'the table' are particular. Here the relation is **diadic** or **two-termed** because the term 'on' relates two terms, *viz.* 'this book' and 'the table.' In the proposition 'three is between two and four' the relating element is 'between' which is universal, while the terms 'two, three, four' are particular. Here we have an example of **triadic** or **three-termed** relation, because the term 'between' relates three terms. Similarly we can have four-termed or **tetradic** relation and so on in more complex propositions. We owe to Russell the formulation of the symmetrical hierarchy of relations which we have exemplified here.

Univocal, Equivocal and Analogous Terms

A term may be used **univocally** or **equivocally.** Univocal and equivocal terms are not two distinct classes of terms, but they represent two different modes of using

terms. According to Mill, "A name is univocal or applied univocally with respect to all things of which it can be predicated in the same sense; it is equivocal or applied equivocally as respects those things of which it is predicated in different senses." Thus man, house, tree are univocal terms, while file, light, foot are equivocal terms. 'File' may mean either a steel instrument or a line of soldiers. 'Foot' may mean either infantry or a unit of measurement. Owing to equivocal use of terms many fallacies are committed. In logic we must always avoid equivocation and must use a term in the same sense throughout a discourse.

An **analogous** term is predicated of two things not in the same sense but in similar signification, *e.g.* brilliant light, brilliant man; a high mountain, a high sound; the foot of a man and the foot of a mountain. In the first pair of examples the term 'brilliant' is used almost equivocally. The point of similarity is that in the case both of light and of man the term 'brilliant' signifies some excellence. When we speak of a foot in reference to a man and to a mountain, analogy is to be sought in the fact that 'foot' implies that on which something stands. Owing to the metaphorical use of language we often reason falsely.

Simple and composite terms.

The distinction between **simple and composite** terms is nothing more than the distinction between single-worded and many-worded terms. A single-worded term is simple, *e.g.* John, house, library, etc., while a many-worded term is composite, *e.g.* the House of Commons, the King of England etc.

Concluding Remarks

In discussing terms we have always borne in mind that a term on its subjective or epistemic side stands for an idea or notion, while on its objective or constitutive side it stands for some object or objects, and also that a term is not a unit of language only but that its logical importance consists in its being a part of a proposition, of which it is an element. We have not discussed the distinction between connotative and non-connotative terms in this chapter, since the importance of the topic requires, as it appears to us, a more comprehensive discussion in a separate chapter.

CHAPTER II

INTENSION AND EXTENSION, CONNOTATION AND DENOTATION, CONNOTATIVE AND NON-CONNOTATIVE TERMS

The Meaning of Intension and Extension

Logicians have used the terms intension, connotation, intent, comprehension, depth, implication and force all in the same sense. Similarly they have regarded the words extension, denotation, extent, sphere, breadth, application and scope as synonymous. The terms that are in most general use are intension and connotation, extension and denotation. Before Mill the words intension and extension were in vogue, but Mill preferred connotation and denotation in view of the convenience of the corresponding verbs 'connote' and 'denote.'

A term denotes things and connotes attributes.

The meaning of every general concrete name has two aspects. On the one hand it indicates a number of objects, on the other it implies certain attributes which are common to those objects. These two aspects of the meaning of a name may be referred to by the employment of the words **Extension** and **Intension.** So it is said that a term **connotes** attributes and **denotes things.** According to Mill a name denotes objects directly and connotes attributes indirectly. But this view cannot be approved, since a number of things are referred to by the same name because they possess some common attribute or attributes.

The use of the terms intension and connotation in the same sense has resulted in a good deal of confusion in logic. There has been, in consequence of this, a difference of opinion among logicians as to whether certain terms are connotative or not. Keynes uses the term intension in its general implicational sense and points out that a distinction should be drawn between the various senses in which the term intension has been used by logicians. A name, he points out, may have **conventional intension**, which is synonymous with **connotation** in the sense in which the term connotation was used by Mill. Besides conventional intension, a name may have **subjective intension** as well as **objective intension** or **comprehension**.

Keynes distinguishes between conventional intension, which is connotation, subjective intension and objective intension or comprehension.

For the progress of science it is necessary that terms should have fixed meaning and that their meaning should be fixed by convention or agreement. According to Welton, if the intension of a term is not fixed, language will cease to be capable of the communication of anything like exact thought. So Bosanquet says: "Surely the question for logic is never what a name means for you or me, but always what it ought to mean." He also says that identical reference is the root and essence of the system of signs called language.

The necessity of fixing the meaning of a term by convention.

What then do we mean by conventional intension or connotation, subjective intension and objective intension or comprehension? Those attributes which are by

The distinction between conventional intension, subjective intension, and objective intension explained by illustration.

agreement supposed to be essential attributes possessed in common by all members of the class of objects denoted by a name form the **conventional intension** or **connotation** of that name. Thus animality and rationality are the conventional intension or connotation of the term 'man', since these attributes have been accepted conventionally as the essential attributes of man. But **subjective intension,** unlike conventional intension, is not fixed but variable. A term may suggest a different set of ideas to different men or to the same man at different points of time. Thus the term 'rose' may suggest to an ordinary man a set of ideas or attributes different from those which the same term will suggest to a botanist. The subjective intension of a term is its intension as it appears to a particular subject or subjects. From this point of view even a proper name has intension, because it is associated with certain attributes in the mind of some persons. Thus to a mother the name of her son suggests various attributes. We may further draw a distinction between conventional intension and subjective intension on the one hand and **objective intension** or **comprehension** on the other. The objective intension of a term consists of all those attributes, known and unknown, which are possessed in common by a number of things denoted by the same name. Thus besides animality and rationality men have many other common attributes, such as corporeity, the capacity to move, animal life, human form etc.; all these attributes possessed in common by men make up the objective

intension or comprehension of the term man.) We may note that from the logical point of view subjective intension is not important.

The above distinction recognised by other Logicians.

Sigwart's distinction between empirical, metaphysical and logical concepts corresponds to Keynes's distinction between subjective intension, objective intension and conventional intension. Bosanquet's distinction between the objective reference of a name and its content for the individual mind corresponds to the distinction between conventional intension and subjective intension. Logicians have used the term connotation in the sense of subjective intension and objective intension as well as of conventional intension indifferently, and there has been a good deal of unnecessary controversy owing to the failure to understand properly the meaning of connotation.

Distinction between extension, denotation and exemplification.

A distinction may also be drawn between extension and denotation. (By the **extension** or **subjective extension,** as Keynes puts it, of a name we mean all the things, real or fictitious, to which the name may be applied. Thus the subjective extension of the term 'man' signifies all the things, real or fictitious (such as the characters of fiction etc.), to which the name may be applied. But the **denotation** of a term consists of all those real things to which the name may be applied. Thus the denotation of the term 'man' consists of all human beings, dead, living and to be born, to which the name 'man' is applicable. If this distinction is borne in

mind we shall understand that such terms as 'centaur', 'demigod', 'winged horse' have no denotation though they have subjective extension. The exemplifications of a term are not co-extensive with its denotation. To exemplify the term 'man' we need only adduce a few typical instances. Exemplification thus excludes many individuals to which the name in question may be properly applied. The denotation or objective extension of a term always has reference to a particular universe of discourse.'

Keynes makes the above distinctions clear by the following apposite example. A metal is defined as an element which may replace hydrogen in an acid and form a salt. This definition of a metal constitutes the connotation of the term metal. All those things, such as gold, silver, iron, copper, brass etc., which have the above property constitute the denotation of the term metal. Thus the denotation of a term is determined by its connotation. The comprehension of the term metal consists of its connotation together with other properties which are also common to all metals, such as fusibility, metallic lustre, a high degree of opacity and the property of being good conductors of heat and electricity, etc. We may however exemplify metals by citing only a few instances, such as silver, iron and copper, which possess all the attributes common to metals. Here silver, iron and copper are exemplifications of the term metal. Similarly a triangle is defined as a plane figure bounded by three straight lines. This definition of a triangle constitutes its connotation. The denotation of the term triangle signifies all those figures to which the name triangle is applicable. Its comprehension consists of such proper-

ties as being a plane figure bounded by three straight lines, having three angles which are together equal to two right angles, and having every angle less than two right angles. We may exemplify triangle by pointing to a few triangular figures. We may note that whether we begin with the connotation or with exemplifications of a term we can arrive at its comprehension and denotation.

Inverse Relation between Denotation and Connotation

The proposition that the denotation and connotation of a term vary in an inverse ratio to each other cannot be accepted in its mathematical form

We are now in a position to understand how far the proposition that the denotation and connotation of a term vary in inverse ratio can be maintained. This doctrine means that if the connotation of a term is increased or decreased its denotation decreases or increases in proportion, and if its denotation is increased or decreased, its connotation proportionately decreases or increases. This statement cannot be accepted in its mathematical form. We can speak of doubling or halving the denotation of a term but it is nonsense to speak of halving or doubling its connotation. Bosanquet points out that when we add to the connotation of the term 'man' the attribute 'white,' its denotation decreases much less than when we add to the connotation of the term 'man' the attribute 'red-haired,' since there are many more white men than red-haired men. If this is true then with the increase of the connotation of a term its denotation does not decrease proportionately.

If we neglect mathematical accuracy the proposition that the denotation and connotation of a term vary inversely holds good within certain limits. If we enhance the connotation of the term 'triangle' by adding the property of having three equal sides, its denotation diminishes, since we then have to omit those triangles, scalene and isosceles, which have not three equal sides. Again if we increase the connotation of the term 'mountain' by adding the property 'Asiatic,' we diminish its denotation, since in this case we exclude non-Asiatic mountains. Similarly the denotation of 'white man' is less than the denotation of 'man'. Again if we take a term whose denotation includes both men and all the lower animals, its connotation will be less than that of the term 'man,' for it will not include rationality. Again if we enhance the denotation of the term 'equilateral triangle' by adding to it all other triangles which are not equilateral, its connotation diminishes, since we have to omit the attribute of having three equal sides. Again if we enhance the denotation of the term 'Indian' by adding to it all men who are devoid of Indian nationality, then we diminish its connotation by omitting the attribute of having Indian nationality.

Within certain limits the denotation and connotation of a term vary inversely.

But it is not always true that if we increase the connotation or denotation of a term, its denotation or connotation decreases. Thus if we increase the connotation of 'triangle' by adding the attribute of having its three angles

The limits of the law of variation.

equal to two right angles, its denotation does not diminish. So also if we enhance the connotation of the term 'man' by the addition of the attribute 'mortality,' its denotation does not decrease. Again if the denotation of 'river' increases by the birth of new rivers, its connotation does not diminish. The birth or death of every child does not decrease or increase the connotation of the term 'man'. Has the community less connotation than the individual? asks Bradley. Bosanquet also holds that the connotation of the term man is not less than the connotation of the term Frenchman or Englishman, since humanity includes 'Frenchmanity,' 'Englishmanity' etc. To limit the proposition that the denotation and connotation of a term vary inversely, Welton states, "The idea of an opposite variation of connotation and denotation is only applicable to classes which can be arranged in a series of varying generality, so that each smaller class forms a part of the next larger; such as figure, plane rectilineal figure, plane triangle, plane isosceles triangle."

Keynes's four laws determining the relation of inverse variation between the intension and extension of a term, which amend the traditional view.

In the light of Keynes's distinction between comprehension and connotation and between denotation and exemplification we may understand the following propositions which he proposes in order to amend the traditional doctrine that the connotation and denotation of a term vary inversely. (1) "Let connotation be supposed arbitrarily fixed, and used to determine denotation in some assigned universe of discourse. Then it will not be true that connotation and denotation

will necessarily vary inversely." (2) "If the connotation of a term is arbitrarily enlarged or restricted, the denotation in an assigned universe of discourse will either remain unaltered or will change in the opposite direction." (3) "If the exemplification (extensive definition) of a term is arbitrarily enlarged or restricted, the comprehension in an assigned universe of discourse will either remain unaltered or will change in the opposite direction." (4) "Any arbitrary alteration in either intensive definition or extensive definition which results in an alteration of either denotation or comprehension will also result in an alteration in the opposite direction of the other." (By intensive definition Keynes means connotation and by extensive definition exemplification.)

Connotative and Non-connotative Terms

The division of terms into **connotative** and **non-connotative** goes deepest into the nature of language, because the essence of every name is its significance. It is wrong, according to Mill, to regard non-connotative names as absolute, since connotative names may also be regarded as absolute. "A non-connotative term is one which signifies a subject only or an attribute only. A connotative term is one which denotes a subject and implies an attribute" (Mill).

Mill's definition of connotative and non-connotative names.

Before we consider which terms are connotative and which are non-connotative, we must remember that every concrete term has subjective intension or comprehension, though it may lack connotation, that is, fixed content. We must also bear in mind

that the denotation of a term, like its connotation, is a part of its significance.

The terms that are connotative.

Following the above we may now examine which terms are connotative and which are not so. Every general name, whether concrete or abstract, is connotative. Thus the term 'tree' applies to a number of things and implies certain attributes to the possession of which the applicability of the term 'tree' is due. Similarly the term 'army,' which is a general collective name, is connotative because it can be applied to a number of groups and also implies certain attributes which every army possesses. All adjectives are connotative. The term 'white' applies to a number of objects such as milk, snow etc. and connotes the attribute whiteness. Similarly good, bad, green, yellow etc. are connotative. Though according to Jevons all abstract names are non-connotative, we may reasonably regard general abstract names as connotative. The term 'virtue' denotes justice, veracity, benevolence, kindness, courage, temperance etc. and it connotes the attribute of goodness. Similarly the term 'colour' denotes redness, greenness, whiteness, etc. and connotes the power of affecting the eye in a particular way. So also the term 'figure' indicates different figures such as triangle, square, circle etc., and connotes the attribute of possessing shape and extension. Connotative are all significant singular names, such as 'the present Speaker of the House of Commons,' 'the present President of the Indian National Congress,' 'the Calcutta Imperial Library'. Certain proper names which have ceased to be proper names

and have become general are connotative; *e.g.* when we speak of a Solon, a Caesar, a Napoleon, a Daniel, such terms have both denotation and connotation. According to Mill such terms as the sun, the moon, the earth, the solar system, God, etc. are connotative. But according to Bosanquet such terms are equivalent to proper names, as their purpose is to indicate some object and not to imply any attribute. We may however say that these terms fall mid-way between connotative and non-connotative names. Such terms as 'the French revolution,' 'the falls of Niagara,' 'President Roosevelt' may also be similarly treated.

The terms that fall mid-way between connotative and non-connotative names.

There are certain terms which can be properly regarded as non-connotative, since they lack either denotation or connotation. Singular abstract names such as whiteness, length, visibility, triangularity, humanity etc., denote according to Welton attributes only and do not connote anything, and may properly be regarded as non-connotative. Again such terms as centaur, winged horse, the king of France in 1930, etc., imply attributes but have no denotation as they cannot be applied to any really existing object. They have subjective extension but no denotation.

Certain terms non-connotative.

Are proper names non-connotative? Logicians have differed in their opinion as to whether proper names have connotation or not. This is due to the failure to distinguish between subjective intension, connotation and comprehension. Proper names certainly have subjective intension, because

Discussion as to why proper names should be regarded as non-connotative.

such a term as 'John' may suggest various attributes to those who are acquainted with John. The term 'John' has comprehension as well, because the object indicated by it does possess attributes known and unknown. Yet every proper name is non-connotative because it lacks fixed content, that is, conventional intension. But Jevons argues that proper names have connotation, and that such a term as 'John Smith' implies 'Teuton' and 'male'. But Keynes points out that the term 'John Smith' may be the name of a race-horse, and it may even be the pseudonym of a woman, just as George Eliot was. The name Jacob originally meant a supplanter, but a man may be called Jacob even if he lacks that attribute. Bosanquet argues that Christian names imply sex and surnames, descent and family relationship. But he admits here that it is self-contradictory to hold that proper names, which have individual reference, have content, which is universal. So according to him the purpose of a proper name is applicative and not implicative. It has no fixed content, he admits, and therefore we can regard it as non-connotative, though it has both subjective intension and comprehension. Even surnames are no longer connotative, because they can be changed at will. Such terms as 'the year 1924,' 'that', 'here', 'now' and every term which refers to some particular point of time or some particular point in space, may be properly regarded from the logical point of view as proper names and are therefore non-connotative. (See Ch. I, discussion on Proper names).

CHAPTER III

THE CATEGORIES OF ARISTOTLE AND THE DOCTRINE OF PREDICABLES. VERBAL OR ANALYTIC, REAL OR SYNTHETIC, AND FORMAL JUDGMENTS

The Doctrine of Categories

To Aristotle we owe the doctrine of **Categories** or **Predicaments** and of **Predicables**. The Aristotelian categories are ten in number and according to him they are a classification of beings. But logicians have differed as to the nature of this classification. According to Mill the classification of categories was intended by Aristotle and the Schoolmen to give us a list of nameable things. According to Bain this classification is not a classification of things but is a classification of predicates. According to Mansel it is a grammatical classification based upon parts of speech such as substantive, adjective, verb and adverb. According to Baynes it is a metaphysical classification, that is, a classification of the aspects of real beings. Welton regards this classification as one of relations conceived by the mind to interpret reality. Thus we find that there is a good deal of controversy as to why Aristotle classified categories and what is the nature of the classification. Joseph appears to be right when he says that "the categories present a logical, but they present also a real distinction, that is, a distinc-

Different views as to the nature of Aristotle's classification of categories. What really is the nature of Aristotle's classification?

tion in the nature of the reality about which we think as well as in our manner of thinking about it". Categories, therefore, are a classification of things as well as of predicates. This will become clear in the light of subsequent discussion.

Aristotle gives us a list of **ten categories**, that is, a list of things and properties, which can be predicated of an individual subject. (There is a distinction recognised by Aristotle and the Schoolmen between the **essential** and **accidental** properties of a thing—to be Socrates is not necessarily to be condemned to death, but to be Socrates is necessarily to be a man.) From such reflections the distinction between categories arose. We may now explain the ten categories, with illustrations which will throw light upon what has been said above. (1) The first category is **Substance**. Aristotle distinguishes between **first** substance, which is an **individual**, such as Plato, and **second** substance, which is a **class**, such as man. The second substance stands for the essential nature of a thing and is a predicate. It is what some logicians describe as a class-name. Substance therefore indicates the essential nature of a thing but attributes do not. All other categories presuppose or are incidental to substance. The proposition 'Socrates is a man' has for its predicate substance, since it gives us the essential nature of Socrates. Socrates cannot but be a man. (2) The second category is named by Aristotle **Quantity**; *e.g.*, Socrates is five feet in height. Quantity is the measurable amount of a thing—five seers in weight, thirty miles away, etc., are terms which give us

Explanation of Aristotelian categories by illustrations.

quantity. (3) The third category or predicament is **Quality.** We describe a thing by mentioning its qualities: *e.g.*, Socrates is ugly. Such terms as fair, good, white, beautiful etc. belong to the category of quality. (4) **Relation** gives us the relation of one thing to another thing; *e.g.*, Socrates is the teacher of Plato. Father, son, wife, husband, like, unlike etc. are terms which belong to the category of relation. We have an example of the category of relation when a relative term is used as a predicate. (5) The next category is **Action.** It tells us how a thing **behaves,** *e.g.*, Socrates is talking. Cutting, running, fighting etc. illustrate the category of action. (6) The sixth category is **Passion.** This is the opposite of action and tells us how a thing is **acted upon,** *e.g.* Socrates is condemned to death. To be hated, to be killed, to be thrown etc. give us the category of passion. (7) **Place** is another category. It tells us **where** a thing is; *e.g.*, Socrates is in his room. Here, there etc. are included in the category of place. (8) **Time** is the next category. It tells us **when** a thing is; *e.g.*, Socrates is talking to young men at midday. Now, then, at dawn, in the evening etc. fall under the category of time. (9) We have as our next category **Situation.** It tells us **how a thing is placed** relatively to another; *e.g.*, Socrates is lying on his bed or sitting in his chair. Such terms as upside down, horizontal, etc. also illustrate the category of situation. (10) The last category is **State.** 'Socrates is dressed' gives us an example of this category. It tells us **in what condition** a thing is. To be armed, to be shod etc. are examples of this category.

Aristotle's classification of categories has been criticised by logicians in different ways. Some have regarded

this classification as useless and irrational. According to Mill, Aristotle's classification of categories is both inadequate and redundant. It is like the classification of animals into men, quadrupeds, horses, asses, ponies etc., that is, the categories are not mutually exclusive but overlap. Therefore the Aristotelian classification is redundant. It is also inadequate, because among nameable things states of consciousness have not been included. Further all the categories except the category of substance can be reduced to the category of attribute. Regarding the Aristotelian list of categories, therefore, as defective, Mill gives a list of three categories to cover all nameable things, *viz.* (1) Feeling (including sensation, thought, volition and emotion); (2) Substance—(*a*) Mind, (*b*) Body; (3) Attribute (including quantity, quality and relation).

Mill's objections to Aristotelian classification of Categories, and his own classification.

Mill's classification is open to grave objections. He regards substance as the permanent possibility of sensations. If so, his two substances, body and mind, so far as our knowledge goes, are nothing but sensations and therefore the category of substance can be reduced to the category of feeling. Again Mill admits that quantity, quality and relation, which are included in the category of attribute, are known through sensation or a series of sensations. If so, the category of attribute can also be reduced to the category of feeling. So if we follow Mill's view of categories to its logical consequence we have but one category, *viz.* feeling. Besides, when Mill holds that the Aristotelian categories are intended to be a

An examination of Mill's list of Categories.

list of nameable things, he falls into error. According to Aristotle categories are really a classification of predicates, though the Schoolmen have interpreted them as a classification of nameable things.

Kant's criticism of Aristotelian categories is based upon misconception.

Kant misreads Aristotle's purpose in the classification of the categories when he says that Aristotle has confused in his classification sensible and intelligible forms, which he has given in the same list without distinguishing between them. Aristotle's purpose was not to distinguish between sensible and intelligible forms. He was concerned with existence and has given us a list of terms which can be predicated of an individual subject.

Some legitimate objections to Aristotle's classification of categories.

We may however observe that Aristotle's classification of categories is open to some legitimate objections. He does not classify categories according to any principle. He has taken a concrete individual and has named in a rough manner what can be predicated of such a subject. So his categories are, as Joseph puts it, forms of individual reality. Aristotle was perhaps aware of the imperfection of his classification; for though he fully discusses the categories of substance, quantity, quality and relation and also treats at some length the categories of time and place, he disposes of the other categories summarily. (We may also point out that all categories are ultimately reducible to three main categories, *viz.* substance, quality and relation. According to Welton and Monahan, "Time and place should not be trea-

ted separately from relation, nor activity and passivity from state."

Categories proposed by subsequent philosophers.

The doctrine of categories proposed by Aristotle has largely influenced subsequent thought, and different philosophers have given different lists of categories. We cannot, within the limits of this book do more than name some of these classifications. The Stoics gave a list of four categories, *viz.* substrate, property (essential), quality (unessential) and relation. Descartes and Spinoza give us a list of three categories, *viz.* substance, attribute and mode. Locke's categories are substance, mode and relation. Kant's categories are twelve in number; they are forms of understanding, *viz.* (1) unity, (2) plurality, (3) totality, (4) reality, (5) negation, (6) limitation, (7) substantiality, (8) causality, (9) reciprocity, (10) possibility and impossibility, (11) existence and non-existence, (12) necessity and contingency.

The Doctrine of Predicables

Categories and predicables.

" Aristotle's list of categories was concerned with a classification of things independent of any relation in which they may stand to other things, while his list of **predicables** was concerned with a classification of attributes as related to a subject." (Welton and Monahan). If we understand the meaning of a term we can at once say to which category it should be referred. We know that the term 'beautiful' belongs to the category of quality whether it is related to any definite subject or not. But we cannot determine to what predicable a

term belongs without knowing its relation to some given subject. Thus predicables are a classification of terms as determined by their relation to some given subject.

totle provides a list of five predicables, viz: Genus, Definition, Differentia, Proprium and Accidens. The traditional list of predicables is due to Porphyry, whose classification differs somewhat from that of Aristotle. According to him the predicables are genus, species, differentia, proprium and accidens (accidens may be either separable or inseparable). A **genus** is a class which has under it sub-classes which are its **species.** Thus under the genus animal there are sub-classes or species such as man, dog, horse etc. Genus, according to Aristotle, can be predicated either of a species or of an individual. Thus we can say that 'men are animals' or 'Socrates is an animal'. When the predicate analyses the essential attributes of the subject term, we have **definition** as the predicate. In the proposition 'man is a rational animal' the predicate is a definition because it unfolds the connotation of the subject term. According to the modern view, when the subject is a singular term, we cannot have definition as its predicate, because an individual cannot be defined. **Differentia** is the peculiar attribute of a species which differentiates it from other things or species. Thus in the proposition 'man is rational' we have a differentia as its predicate, because rationality marks off man from other species, such as horse, dog etc. A **property** (proprium) follows from the

The list of predicables given by Aristotle and Porphyry, and explanation of Aristotelian predicables by examples.

definition of the subject, just as an effect follows from its cause or a conclusion follows from its premises. The definition of man gives us two attributes, *viz.* animality and rationality; the former provides the general nature of man and the latter his specific nature. Now in the proposition 'man is mortal' mortal is a property which follows from the generic nature of man, *viz.* animality. Again in the proposition 'man is a tool-using animal,' the predicate is a property which follows from the specific nature of man, *viz.* rationality. While a property follows from the essential nature of a thing and is necessary to it, (2) **accidents** (accidentia) are not indispensable to a thing and do not follow from its essential nature. Thus in the proposition 'some men are fair' we have as our predicate an accident (accidens), because to be fair is not of the essential nature of man nor does it follow therefrom. 'Socrates is armed' is a proposition which has as its predicate an accident, because to be armed is not indispensable to Socrates. Aristotle holds that in predication what applies to a class (which according to him is the 'second substance') also applies to the individual, the 'first substance' included under the class.

Some remarks on the Aristotelian predicables.

The distinction between definition and property is arbitrary. Why should we not regard mortality as an essential attribute of man like animality and rationality? Why should we regard mortality as a property (proprium) of man? Moreover, Aristotle does not clearly distinguish between genus and species, as Porphyry does. Aristotle classifies things into natural kinds according to

the difference in their essential nature. Thus man, horse, monkey etc. are natural kinds, because each class differs from the other in essence. But the theory of evolution proves that between two classes there is no essential difference and one class may easily pass into another. Aristotle's first three predicables are based upon the essential nature of things, while properties follow from the essential nature of things but accidents have no relation to the essence of a thing. Genus, definition, differentia and property are necessarily related to one another, and upon them scientific classification is based. But accident is not causally related to other predicables. Unlike the categories, the five predicables are exclusive and not overlapping. When the subject and the predicate of a given proposition agree in denotation and connotation, we have a definition as the predicate ; *e. g.* 'triangles are rectilineal plane figures bounded by three straight lines.' When the subject and the predicate of a given proposition agree in denotation but not in connotation we have a property, which follows from the differentia of a class-name, as the predicate ; *e. g.* 'men are tool-using animals.' When the subject and the predicate of a given proposition agree partially in connotation but differ in denotation we have a genus as the predicate ; *e.g.* 'man is an animal.' When the subject and the predicate of a given proposition differ both in denotation and in connotation we have an accident as the predicate ; *e.g.* 'some men are black'.

We have already stated Porphyry's classification of predicables. Before explaining its logical character we

shall show the distinction between genus and species as Porphyry draws it. We may at the outset remark that Porphyry failed to appreciate Aristotle's purpose in classifying predicables. Aristotle wanted to classify predicates according to the relations existing between them when they are referred to the same subject. Porphyry gives an account of the ways in which predicates may be referred to the individual subject, which Aristotle ignored. Thus Aristotle's list is based upon the relations between predicates of the same subject, while Porphyry's list provides an account of the modes of predication. Porphyry substitutes species for definition because an individual cannot be defined: only a class may have definition. A genus according to Porphyry is a natural kind or class which can be subdivided into subordinate natural classes, which are the species of the higher class. Thus 'triangle' is a genus while equilateral, isosceles and scalene triangles are species of it. A term which is a genus in relation to one term may be a species in relation to another term. Thus 'triangle' is a genus in relation to equilateral, isosceles and scalene triangles, but it is a species in relation to 'rectilinear figure'. Again 'animal' is a species in relation to living objects, but it is a genus in relation to man, monkeys, elephants etc. Thus the terms genus and species are correlative terms, since the meaning of each can be understood only by understanding the meaning of the

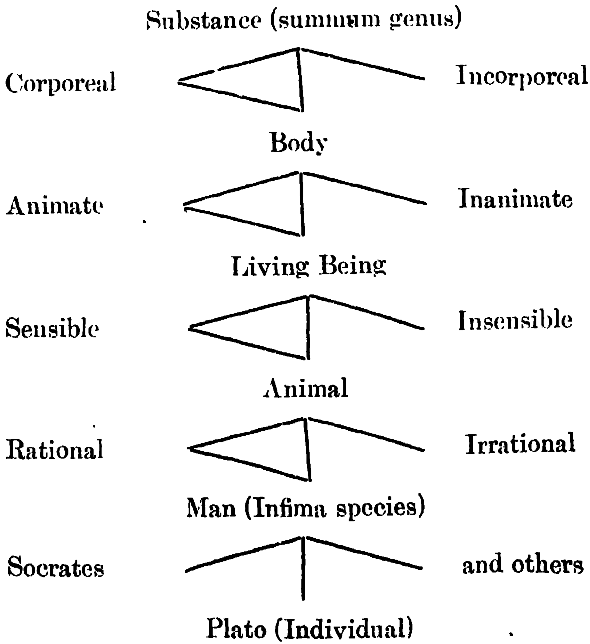
Difference between Aristotelian list of predicables and that of Porphyry.

Explanation of the terms genus, species, cognate species, cognate genus, subaltern genus or species, summum genus, infima species, real kind.

other. Species which are co-ordinate and belong to the same genus are called **cognate species.** Thus in one of the above examples man, monkeys, elephants, and in the other equilateral, isosceles and scalene triangles are cognate species. Every genus which is above a given species is a **cognate genus.** Thus each of the terms triangle, rectilinear figure, figure is a cognate genus of each of the terms equilateral triangle, isosceles triangle and scalene triangle. The next higher class of a species is called its **proximum** or **proximate genus.** Thus triangle is the proximum genus of equilateral triangle and animal of man. Every genus or species which can function both as a genus and as a species is called a **subaltern** genus or species. Thus the term 'animal,' which is a genus in relation to man and a species in relation to living things, may be called either a subaltern genus or a subaltern species. A **summum genus** is a class which cannot be subordinated to any other higher class; that is, a summum genus can never be a species. An **infima species** is a class which cannot be subdivided into subordinate natural classes, but can only be divided into individuals; that is, an infima species can never be a genus. Man, though it may be subdivided into mathematicians, poets, politicians etc., is an infima species because these subdivisions are not natural classes and therefore are not species. The division of man even into male and female is not regarded by traditional thinkers as a division into natural classes. Man can be divided naturally only into individuals. To state the character of a natural class or a **real kind,** that is, of a genus or species, Mill says, "Every class which is a real kind, that is,

which is distinguished from all other classes by an indeterminate multitude of properties not derivable from one another, is either a genus or a species." Aristotle recognised ten summa genera, which are his categories. Modern logicians recognise but one summum genus, *viz.* substance. But for practical purposes we may recognise several summa genera. Thus to sociologists man is the summum genus, to chemists material substance and to biologists life. We may illustrate the above by the **Tree of Porphyry** or Ramean Tree (so named after Ramus who gave prominence to it) :—

Porphyry's Tree, that is, his predicamental line.



Explanation of Porphyry's list of predicables by examples.

When a general term is predicated of another general term, the predicate is a genus, *e.g.* 'man is an animal' or 'a triangle is a rectilinear figure'. When a general term is predicated of an individual term we have a species as the predicate, *e.g.* 'Socrates is a man' or 'Socrates is an animal.' "The excess of the connotation of a species over that of its proximate genus is called the differentia or difference of that species" (Welton). Thus in the judgment 'man is rational' we have as predicate the differentia of man, that is, the attribute which distinguishes men from other cognate species. Again in the proposition 'isosceles triangles have two equal sides,' we have a differentia as the predicate. The distinction between differentia and property (proprium) is founded upon conventions of language rather than on the nature of things. We have already stated that a property follows from the essential nature of thing. Thus in the propositions 'man is mortal,' 'man is a cooking animal', the predicates illustrate what differentia is. In this regard Porphyry is at one with Aristotle. An accident can be removed from a class or an individual without affecting it fundamentally. Here also Porphyry's account of accidents is not different from that of Aristotle. But the former makes a distinction between separable and inseparable accidents. In the propositions 'All European ruminants are cloven-footed', 'Gandhi is an Indian,' 'Socrates had a snub-nose,' 'Alexander is the son of Philip,' the predicates are inseparable accidents. In the propositions 'some men are red-haired,' 'Mr. MacDonald is the Prime Minister of England,' we have separable accidents as predicates.

We may now remark that neither Aristotle's list of predicables nor that of Porphyry considers the possibility of a singular term functioning as a predicate, as in the sentence, 'Calcutta is the capital of Bengal.' Besides, we may ask why we should distinguish between property and inseparable accident from the logical point of view, since in both cases the attribute predicated is inseparable from the subject ?

Some remarks on predicables.

Verbal, Real and Formal Judgments

It will be convenient at this stage to treat of the distinction between verbal and real propositions or judgments and also to explain what a formal proposition is. The terms **verbal, analytic, essential** are used in the same sense, while the terms **real, synthetic, accidental** and **ampliative** are synonymously used when they are employed to qualify propositions. The distinction between analytic and synthetic or verbal and real applies only to affirmative propositions. A truly verbal proposition is a purely synonymous one, and cannot be regarded as the same as an analytic proposition in which the predicate asserts some essential attribute or attributes of the subject. Such propositions as 'Cicero is Tully' and 'an ass is a donkey,' 'wealth is riches' etc. are purely verbal propositions. But the term verbal proposition has been used by logicians to indicate essential or analytic proposition as well. We have already noted the distinction between the essential attributes of a thing and its accidental attributes. An essential or analytic proposition is one in which the predicate asserts some essential

The nature of verbal, analytic or essential propositions.

attribute of the subject. Thus the propositions 'man is rational,' 'man is a rational animal,' 'man is an animal' are essential, because in each case the predicate asserts some essential attribute or attributes of the thing denoted by the subject term. Similarly the proposition that a square is a rectilinear figure bounded by four equal sides' is a verbal proposition. In other words, an essential or analytic proposition asserts some attribute, or all the attributes, connoted by the subject. According to Mill such a proposition is useless because it does not impart any new information about the subject and is not capable of proof. As soon as we know the subject term we know its essence and we require no proposition to state it. It has therefore been stated that the predicate of an analytic or essential proposition must either be a genus or a species or definition or a differentia. But we may here remark that because proper names imply no quality such a proposition as 'Socrates is a rational animal' should not be regarded as an analytic proposition. The Aristotelians regard it in this way, because Socrates has no connotation and therefore cannot be defined. Such a proposition should properly be regarded as accidental, since the predicate in this case states something which is not implied by the subject term ; but when the subject is a singular significant term we may have an analytic proposition, as in the case 'this great philosopher is a man,' because to be a philosopher is to be a man, that is, the subject implies the predicate. Thus "a verbal proposition is one which gives information only in regard to the meaning or application of the term which constitutes its subject," as Keynes puts it.)

The character of a real, accidental or synthetic proposition.

While an analytic proposition affirms some part or the whole of the connotation of the subject, a real, synthetic or accidental proposition affirms attributes accidental to the subject, not essential to it. Thus such propositions as 'men are cooking animals,' 'some men are red-haired,' 'some Indians are industrious,' 'a square is a rectilineal figure having its sides and angles equal' etc. are accidental or synthetic propositions. Such propositions were called accidental by the Schoolmen because they assert some attribute not essential to the subject, that ~~is, accidental to it.~~ So in a real or ~~synthe~~tic proposition the predicate is either a property (proprium) or an accident.

Bradley's view as to the distinction between analytic and synthetic judgment.

Bradley holds that extension and intension are subjective, that is, a term has different meanings to different persons. If this be true, then the proposition 'man is rational' is analytic to those who know that to be rational is the essential nature of man, but the same proposition is synthetic to those who do not know that the term 'man' implies rationality. Similarly the proposition 'a triangle is a figure bounded by three straight lines' may be either synthetic or analytic according to the knowledge of different men. So Bradley holds that if extension and intension are relative to our knowledge, it is fatal to the Kantian distinction between analytic and synthetic judgments. He also remarks that a synthetic judgment is an analytic judgment in the making. This means that as soon as we know that an attribute belongs to a subject we have an

analytic judgment. If we do not know that rationality is an essential attribute of man, then the judgment 'man is rational' is a synthetic judgment for us. But as soon as we know that rationality is an essential attribute of man, the judgment becomes analytic for us.)

Criticism of Bradley's view. The distinction between analytic and synthetic judgment defensible.

(In the previous chapter we have shown that the connotation of a term is fixed by convention and is not the same as its subjective intension. Logic is concerned with universal truth and not with individual points of view. The proposition 'man is a rational animal' is analytic to everyone, because it has been established by agreement or convention that animality and rationality form the connotation of man and are essential attributes of man. Similarly 'some men are red-haired' is a synthetic or accidental proposition to everyone, because to be red-haired is not an essential attribute of man.) But when a judgment does not assert this content but asserts something else, it is synthetic. We may also observe that with the growth of knowledge the connotation of a term may change, and an analytic judgment may become synthetic. But even then we require the new connotation to be fixed conventionally.

The nature of a formal proposition.

A **formal** judgment is one the truth of which can be estimated by considering the form of it. Such propositions as A is A, No X is not-X, Any X is either P or Q, If all A is B then no not-B is A, A man is a man, etc., are formal judgments. A formal judgment should be distinguished from a verbal judgment because the validity of a formal judgment depends upon

its formality. (The validity of a real proposition depends upon questions of fact, that of a verbal proposition upon mere meaning, while the truth of a formal proposition depends upon its bare form. From the logical point of view formal propositions are very important.)

CHAPTER IV

THE PROBLEM OF DEFINITION

Definition consists in unfolding the connotation of a term. We define a term to make its meaning clear and distinct and to make the progress of thought possible. It has been a subject of controversy whether definition is of names or of things. To us this controversy is meaningless. Names are very often names of real things, and in defining such names we cannot but take cognisance of the fundamental attributes which are connoted by the name. But in some cases the object of definition is to give clearness to the meaning of a name without any reference to real things, and such definitions are definitions of names only. The view of Whately and Mill is defective inasmuch as they hold that all definitions are definitions of names only. Mill argues that geometrical definitions are all ideal. In the real world there cannot be any geometrical point, having position but no magnitude. Similarly he argues that we may rightly define a dragon as a serpent breathing flame, though no such being may exist in the world. We agree with Mill that there are nominal definitions, that is, definitions of names. But there are other definitions which are as much definitions of things as of names. When we define man as a rational animal, we cannot ignore the

General nature of Definition.

Nominal and real definitions.

nature of things to which the term man is applicable. In order to define names which have their objective counterpart in the world of reality we have to take recourse to observation, analysis, comparison, abstraction, generalisation, and we must also be able to distinguish between primary or fundamental qualities and secondary or derivative qualities of things, without which such names cannot be well defined. So our conclusion is that some definitions are **real**, and must be based upon the nature of things ; while others are **nominal**, in which case we have to depend upon usage. Mill also recognises this when he says that though definition is of names, it enters deep into the nature of reality, as in defining justice, virtue etc. Remembering that names are often indissolubly related to things, we may say with Mill that "the simplest and most correct notion of a definition is, a proposition declaratory of the meaning of a word, namely, either the meaning which it bears in common acceptation, or that which the speaker or writer, for the particular purposes of his discourse, intends to annex to it." We may here remark that Ueberweg's view that all definitions are of notions is also one-sided. We define terms, and since terms stand for notions and have objective counterparts, definitions, though of terms, are as much definitions of notions as of objects. According to Welton and Monahan, real definition may be either analytic or descriptive, while a nominal definition may be either ostensive or biverbal. We shall explain hereafter the meaning of these expressions. But at this stage we may distinguish between **substantial** definition and

Substantial and genetic definition.

genetic definition. Terms are defined **substantially** when the essential attributes of the things denoted by the term are stated; they are defined **genetically** or **constructively** when we state how the distinction of the classes denoted by them is effected. Thus in defining the term tree substantially, we state the essential attributes which are common to all trees, but we define the term genetically by describing the considerations as a result of which certain things come to be called by the name tree. But genetic definition is for logical purposes useless.

Implication of the statement that definition should be per genus et differentiam.

We are now in a position to understand what is meant by the Scholastic view, that definition should be *per genus et differentiam*. This means that in defining a species we must state its generic property and also its differentia. Thus we should define man as a rational animal and isosceles triangle as a three-sided rectilinear figure having two sides equal. But Mill holds that in defining a class-name we must not only state its generic property and differentia but we must also state other attributes which are common to all the members of the class. Thus according to him properties and inseparable accidents should also be included in definition. Thus he says that man should be defined as a rational, organised, corporeal and living animal. But such a definition does not observe the law of parsimony. Since life, corporeity, organisation follow from animal nature, we can rightly define man as a rational animal. So in defining a class we should not mention any accidental quality,

but should state its essential attributes, that is, those attributes upon which others are grounded. Thus properties need not be included in a definition, and accidents, whether separable or inseparable, should be omitted in defining a class name. Again, in defining a species we should state its differentia and the nature of its proximate genus and not of some remote genus. Thus to define 'man' as a rational being is untenable, since it does not state the animal nature of man. If by differentia we mean one attribute or a collection of attributes, then it is sufficient to say that definition should be per genus et differentiam, instead of saying with Mill that it should be per genus et differentias. Every definition of a general name must be a universal proposition, that is, it must be expressed in the form 'Every S is P'.

Different kinds of definition, of which the analytic definition alone is scientific.

A scientific definition should be **analytic**, that is, it should state the connotation of the defined term. Thus the scientific definition of a triangle is that it is a rectilinear figure bounded by three straight lines. But various kinds of unscientific or popular definition are recognised. (a) A name whose meaning is unknown is **ostensively** defined by pointing out the thing which it indicates. Thus, pointing to an orange we may say, this is an orange. Recourse is often had to such ostensive definition in the education of children. But this is not definition proper. (b) When the meaning of a name is unknown we may define it by a synonym whose meaning is known. Commiseration may be defined as pity, or Swaraj as self-government. In translating

from a foreign tongue we may be said thus to define **biverbally.** (c) An **extensive** definition consists in defining a term by pointing out certain things which it denotes. Thus to define virtue we may point out that pity and veracity are virtues, or to define mountain we may say that the Himalayas, the Vindhyas, the Alps are mountains. (d) **Descriptive** definition consists in describing a thing by means of some of its unimportant attributes. Thus we describe a landscape by naming some of its features, or a horse by saying that it is an animal beautiful to look at and is used in time of war or for drawing carriages. Of the above kinds of definition logic is concerned with analytic definition only, since it contributes greatly to the advancement of knowledge and is based upon the knowledge of the nature of things. Analytic definition is definition per genus et differentiam.

Some remarks on the Scholastic view of definition that it should be per genus et differentiam.

The Scholastic view of definition, that it should be per genus et differentiam, is indeed important, but it is necessary to make some remarks upon this view of definition. According to the Schoolmen, species were supposed to be unalterably fixed, but the theory of evolution shows that no strict line of demarcation can be drawn between different species, and species and sub-species may be traced to a common origin. Besides, in the course of time what we regard as the differentia of a species may come to be regarded as a mere property of it. Thus properties may become differentiae and differentiae properties. Thus with the growth of knowledge definitions undergo modi-

fication and should properly do so. Discovery and definition should go hand in hand. So no definition of class-names should be regarded as unalterably fixed. Though the object of definition is to clarify the meaning of a term, it should not bar the progress of thought and science. Thus with the progress of knowledge the denotation of a term may increase and its connotation consequently may become vague. But even in such a case an attempt should be made to define properly after due observation. There may be marginal instances between classes, but the attempt should be made to bring them under one class or another. Thus the sponge, which was supposed to be a marginal instance between animals and plants, has been found to be akin to animals and has been included under the class animal. Furthermore, marginal instances should not prevent us from defining a class if such definition serves our purpose.

Limits of definition.

We may now point out the limits of definition. Not all names can be defined: only significant names are capable of definition. Proper names, therefore, which have no connotation, cannot be defined. Similarly, elementary sensations which cannot be analysed can have no definition. Thus feelings of pleasure, pain and anger cannot be defined. We cannot define the sensation of white or red nor can we define the sensation of sweetness. Singular abstract names, such as triangularity, whiteness etc. cannot be defined because such terms are not significant. We can however describe how a particular sensation originates by the stimulation of sense organs. We can also point out what are the constituent elementary

feelings in a complex feeling. But such processes cannot be regarded as definition proper. But a singular significant name can be defined by marking out its fundamental character and omitting inessential attributes. Thus 'the present President of the Indian National Congress' may be defined as the person who is at present the executive head of the Congress. We need not name the other minor attributes which this person possesses. According to the Schoolmen, summa genera cannot properly be defined, since they cannot be brought under higher genera. Thus according to them the Aristotelian categories cannot be defined. Again according to them sub-divisions of an infima species cannot be defined. Thus the term 'negro' cannot be defined. Modern logicians however hold that the term 'negro' can be defined properly as a black man. Similarly such terms as Europeans or Indians may be defined. General abstract terms may also be defined. Thus 'fault' may be defined, according to Mill, as a quality productive of evil or inconvenience, and 'eloquence' may be defined as the power of influencing the feelings by speech or writing.

The Rules of Scientific Definition

A number of rules are given by logicians to guide us in defining properly. They are the following :—(1) Definition must give only the essence or connotation of the term defined. "The essence of anything is that in virtue of which it is such a thing. It is in virtue of being a three-sided rectilinear figure that anything is a rectilinear triangle :

The violation of the first rule leads to either redundant or too narrow or too wide definition.

in virtue of being an institution for the education of the young, that anything is a school : in virtue of having value in exchange, that anything is wealth" (Joseph). If the definition states more than the connotation of the term to be defined, then the definition is redundant and may be too narrow. A definition is too narrow when the denotation of the definition is less than the denotation of the term to be defined. Thus if a triangle is defined as a plane rectilinear figure bounded by three equal straight lines, the definition is too narrow, because it excludes isosceles and scalene triangles. Again to define man as a rational animal having a white complexion is also too narrow a definition, because the denotation of the definition is less than the denotation of the term defined. But if properties or inseparable accidents are included in the definition, the denotation of the definition remains the same as that of the term defined. Thus if we define a triangle as a three-sided rectilinear figure having three angles, we include in the definition the property of having three angles. In this case the denotation of the term defined is the same as that of the definition. Yet such a definition is faulty because it does not observe the principle of parsimony, and also because it suggests that there may be triangles that have not three angles. If we omit some part of the connotation of a term in the definition, the definition is incomplete and becomes too wide, for then the denotation of the definition becomes greater than the denotation of the term defined. Thus if we define a triangle as a rectilinear figure we mention only a part of the connotation of the term triangle, and thus the denota-

tion of the definition, since it includes figures other than triangles, becomes greater than the denotation of the term defined. Similarly the definition of man as an animal is too wide. This rule follows from the nature of definition, which should be per genus et differentiam. From the above it follows that the denotation of a definition should be equal to the denotation of the term defined.

The violation of the second rule leads to obscure definition.

(2) Definition should be clear and should not be expressed in unfamiliar, figurative or ambiguous language. The non-observance of this rule means the explanation of the unknown by the equally or more unknown. The fallacy arising out of the breach of this rule is technically known as 'ignotum per ignotius' or 'per aeque ignotum.' Memory should not be defined as 'the tablet of the mind.' Again such definitions as 'necessity is the mother of invention,' 'man is the crown of creation', etc., are figurative. Again Dr. Johnson's definition of a net as "a reticulated fabric, decussated at regular intervals" is obscure. 'Eccentricity is peculiar idiosyncrasy,' 'fluency is an exuberance of verbosity,' are also examples of obscure definition. (3) The definition of a term should not be by some synonym, that is, a term should not be defined by itself. The violation of this rule leads to tautologous definition, which is technically known as circulus in definiendo. Thus 'wealth is riches,' 'truth is veracity' are examples of tautologous definition. Again, a cause should not be defined as that which produces an effect, nor an effect as that which is produced by a cause.

The violation of the third rule leads to tautologous or circular definition.

Correlative terms can be defined only by defining the relation between them. Again to say that 'pleasure is desired, that which is desired is good, what is good is pleasant, therefore pleasure is good' is an example of circular definition. But synonymous definition is often useful and adds to our knowledge when an obscure term is defined by a synonym which is simple and clear, *e.g.* 'an entrepreneur is the organiser of an industry.' (4) A definition should not be negative when it might be affirmative. 'Virtue is the opposite of vice', 'solid is that which is neither liquid nor gaseous' are examples of negative definitions, and are useless because they do not unfold the meaning of the term defined. But terms which have a negative sense can rightly be defined negatively. Thus we may define a bachelor as an unmarried man, an alien as a person who is not a citizen, or an outlaw as a person who does not receive the protection of law.

The violation of the fourth rule involves the fallacy of negative definition.

Some concluding Remarks

In conclusion we may remark that it is often very difficult to define terms per genus et differentiam because we do not know the connotation of every term. Thus it is not easy to define dog, monkey, elephant etc. So also social sciences such as economics, politics, sociology etc. are not easy to define. We find difficulty also in defining such terms as community, right, duty etc. Our knowledge of things not being perfect and being progressive, we have sometimes to take recourse to descriptive definitions, which often prove useful. In such a definition we mention attributes which are not essential to things. Thus economics is defined as the science that deals with

consumption, production and distribution ; a nation is defined as an organised people having a common nationality and occupying a definite territory, and so on. Again man is defined as a cooking animal or a featherless biped. Such definitions are descriptive. Some descriptive definitions may be very useful, others not so useful. We have also remarked that definition should be as progressive as our knowledge and should not be regarded as stereotyped. Atoms were once defined as hard, ultimate, indivisible particles of matter, but modern physical research has established that the atom is not simple but composite, being made up of protons and electrons.

CHAPTER V

THE DOCTRINE OF DIVISION AND OF CLASSIFICATION THE PROBLEM OF NOMENCLATURE AND OF TERMINOLOGY

Division

Keynes says, "The term Division may be defined as the setting forth of the smaller groups which are contained under the extension of a given term. It is also defined as the separation of a genus into its constituent species." Though division involves the analysis of the denotation of a term, it does not consist in enumerating the individuals belonging to a class. In logical **division** we divide a **higher class** or a **genus** into **sub-classes or species**, which may again be sub-divided into sub-species till the infimae species are reached. The genus which is to be divided is called the **totum divisum** (divided whole) or **dividend.** The species into which it is analysed are styled the **membra dividentia** (dividing members). Thus we divide animals into men, elephants, horses, monkeys etc., and triangles into equilateral, isosceles and scalene. In dividing a genus we think of an attribute which is possessed by some of its members and not by others, and this suggests the *fundamentum divisionis* or basis of division. In the above example of the division of triangles, the principle of division is the relation of the sides. Similarly when we divide triangles into obtuse-angled, right-angled and acute-angled, the division is according to the size of the largest angle. In dividing men into

The general nature of division.

Europeans, Asiatics, Africans etc. the principle of division is the continent in which they live. The above examples show that the same genus may be divided according to different principles, as in the case of dividing triangles according to the relation of their sides and according to the largest angle. If the same genus is divided according to different principles we have an example of **co-division,** and the classes obtained by co-division overlap each other. Thus a right-angled triangle may be either isosceles or scalene.

Progressive division must be gradual and should not make a leap.

In **progressive division** we must proceed gradually from the higher class to the lower species, and we must not jump from a higher class to the lowest species, leaving out the intermediate steps. Division, therefore, must be gradual and step by step. This is expressed by saying 'Divisio non faciat saltum', that is, division must not make a leap. Thus in dividing men we may divide them into Asiatics, Europeans, Americans etc., and again we may divide Asiatics into Indians, Chinese, Persians, Japanese etc., and again we may divide Indians into Hindus, Moslems, Sikhs, Christians, Zoroastrians etc. Such a division is progressive and gradual.

The relation between division and definition. Division presupposes definition.

Division presupposes definition. If we are not aware of the definition of a name, we cannot find out the principle of division. Definition gives us the connotation of a term and it is connotation which determines denotation. We do not get at the connotation of a term by observing individual

instances, but the connotation of a name is given and we bring individuals under a class according to the possession or non-possession of certain attributes, which we know to be the class-attributes. Thus the definition of triangle gives us the connotation of triangle, and in dividing we group triangles into sub-classes according to some principle, but if we are not aware of the connotation of the class triangle, we cannot divide it. Thus division is concerned with concepts or universals. We divide a class into sub-classes, that is, a universal into more determinate universals. We may make the relation between definition and division clear by another example. If we want to divide wealth into its species we must know the definition of wealth, namely, that it is that which has value in exchange. There are things which have value in use and not in exchange, and such things should be excluded from the category of wealth. When we have thus obtained the definition of wealth we can divide it into its species. We therefore find that definition is fundamental, while division is derivative.

The principle of division is not always present in all the sub-divisions.

Should the principle of division be present in all the sub-classes? When we divide a class by contradictories, the principle of division is present in both of its members, as in dividing colour into white and not-white. But the principle of division may not be present in all the sub-classes in all cases of division, as when we divide undergraduate students according to colleges, for there are undergraduate non-collegiate students.

Logical division should be distinguished from physical, metaphysical and verbal division. In **physical** division we divide an individual thing into its constituent parts, as in dividing a ship into mast, hull, sails etc., or in dividing a watch into case, hands, face etc. Chemical division, *e.g.* the division of water into hydrogen and oxygen, is nothing but physical partition. In the case of logical division the genus divided is predicable of each of its sub-classes. Thus X is divided into Xa, Xb, Xc etc. We can say man is an animal, the monkey is an animal, the dog is an animal etc. But we cannot say that the mast is a ship or the hull is a ship. **Metaphysical** division is the mental division of a thing into its attributes, as when we divide gold into yellowness, hardness, malleability etc. Here also the thing divided is not predicable of each of its qualities. We cannot say that yellowness is gold. Again **verbal** division, as in the case of dividing an ambiguous word into its different significations, is not logical division. Thus we distinguish the meanings of 'vice' as either some moral fault or a mechanical instrument.

Logical division should be distinguished from physical, metaphysical and verbal division.

Some logicians wrongly regard division as merely formal. A correct division cannot ignore consideration of facts. Our knowledge of the general attribute does not give us the differentiae of the species, without the knowledge of which a higher class cannot be divided into sub-classes. This knowledge is gained by observation of facts. The definition of

Division is not merely a formal process but it requires a knowledge of facts.

'triangle' does not give us the relation of its sides according to which we divide it into equilateral, isosceles and scalene. Again the knowledge of the connotation of 'animal' does not give us the differentiae of its sub-classes, which can only be learned by a study of facts. Thus it is not correct to say that classification is material while division is purely formal. Moreover, division involves classification. If we want to divide novels we have to enumerate mentally the novels which we have perused.

The Fundamental Rules of Division

Our previous study enables us now to examine the fundamental **rules of division.** They are :

I. "The members of the division shall be mutually exclusive."

II. "Collectively they shall be exactly co-extensive with the class that is divided." This means that the denotation of the sub-classes collectively should be equal to the denotation of the class divided.

III. "Each distinct act of division should proceed throughout upon one and the same basis or principle."

IV. "If the division involves more than one step, it should proceed gradually and continuously from the highest genus to the lowest species, that is to say, it should not pass suddenly from a high genus to a low species." (Keynes).

Though not strictly necessary, another rule may be added, *viz.*—

V. No individual sub-division should be equal in extent to the class divided.

Logical division is impossible without observing the first and second of these rules, which are fundamental.

The non-observation of the first and the third rule may lead to cross or overlapping division.

Rule III does not exclude the possibility of dividing a genus according to different principles. Rule IV requires that division should give us a hierarchy of classes or a graduated series. Non-observation of the first and the third rules may lead to cross or overlapping division. Thus division of animals into invertebrates, fishes, amphibians, reptiles and birds involves cross division. Again if we divide men into white men, negroes, yellow men, Hindus, Mohammedans, Christians, Europeans, Americans, Asiatics, Africans etc., there is cross or overlapping division, because white men may be Europeans or Americans and Christians or non-Christians. But in some cases the non-observation of the third rule may not lead to cross division, as in dividing triangles into isosceles, scalene and equiangular, because equiangular triangles are equilateral. Again the observation of this rule may not prevent overlapping division, as when we divide triangles into equilateral isosceles and scalene, since equilateral triangles are at the same time isosceles. To get rid of this difficulty we should define an isosceles triangle as a triangle which has only two of its sides equal.

The breach of rule II leads to either too wide or too narrow division. If in dividing a class we omit some of its sub-classes, then the collective extent of the sub-classes becomes less than the extent of the class divided, and in such a case there is too narrow division.)

The cases in which the division is either too wide or too narrow.

Thus if we divide rectilinear figures into triangles and quadrilaterals only, or animals into men, monkeys and horses only, there is too narrow division. Again if in dividing a class we mention all the sub-classes and main classes besides, then the collective extent of the sub-divisions becomes greater than the extent of the class divided, and in such a case there is too wide division. Thus if we divide men into Europeans, non-Europeans, monkeys, elephants etc. there is too wide division. Even if the third rule is observed, that is, if we adhere to one principle in dividing, there may be either too narrow or too wide division, for we may omit some of the sub-classes or include some species within sub-classes which are outside the genus divided. The non-observation of the fourth rule may also lead to too narrow division. If we leave out intermediate steps in progressive division, the collective extent of the sub-divisions may be less than that of the class divided. Progressive division is exemplified when a botanist starts with the summa genera of plants, *viz.* exogens, endogens and acrogens, and sub-divides them into varying orders, which again he sub-divides into varying genera and these again into varying species till the infimae species are reached. Such procedure is scientific and helps the development of knowledge.

Division by Dichotomy, or Bifid Division

Dichotomous division is based upon the principles of contradiction and excluded middle. Division by

Division by Dichotomy is formal in character. Some observations against such divisions.

Dichotomy, or dichotomy by contradiction, is the division of a class simply with reference to the presence or absence of a given attribute or set of attributes.) Thus we may divide colour into white colour and not-white colour, or X into XA and XA′, A′ meaning not-A. Such a division, though formally valid, does not represent our actual procedure of division. Besides, in dividing X into XA and XA′ we require the knowledge of one of the terms in the sub-divisions, *viz.* that XA is included in the class X. Dichotomous division is exemplified by the Tree of Porphyry or Ramean Tree. Many logicians eloquently praise division by dichotomy as perfect. Thus Bentham and Jevons are enchanted by the matchless beauty of the Ramean Tree. We however cannot be satisfied with merely formal division, in which the negative term is always indefinite. The following objections can be made against division by dichotomy. The sub-class indicated by the negative term is always indefinite in extent, though we may go on sub-dividing it. XA′ does not tell us what its denotation is. Such a division does not take into consideration actual facts and is thus cumbrous. When a class can be divided into natural sub-classes, it is meaningless to divide it by two contradictory terms. Further such division is hypothetical in character, since we do not know whether the class indicated by the negative term is existent or not. Mill regards such a division as too formal, while Mansel speaks of it as not sufficiently formal. From the practical point of view it is useless. No division is possible without

material knowledge. It does not provide any means by which correct divisions may be effected. But it may be said that except in the case of division by dichotomy we cannot formally find out whether a particular division conforms to the rules of division or not. For material division it is necessary to find out whether the sub-divisions have reference to the universe of discourse in question. But in formal division such reference is not necessary, since dichotomous division is hypothetical in character, the negative term being indefinite. Welton remarks that such a division, if not purely formal, must also be based on fact, as in the case of the division of men into Europeans and non-Europeans if any, Europeans into Englishmen and non-Englishmen if any, and so on.

Purely formal division is useless.

Venn gives us various examples of formal division which for all practical purposes are useless. We may give an example. If we are concerned with three terms S, M, P, we may have the following formal divisions, *viz*. SMP, SMP′, SM′P, SM′P′. S′MP, S′MP′, S′M′P and S′M′P′. Here S′, M′, P′ stand for not-S, not-M, not-P. For other examples of formal division students may consult Venn's *Symbolic Logic*.

Division is very useful from the scientific point of view.

Division is very useful from the scientific point of view. It clarifies our thinking and makes progress in knowledge possible. Every science has to take recourse to division and classification. without which it cannot expect to attain its object.

Classification

We often speak indifferently of division and **classification** in the same sense, but a distinction should be drawn between them. We have already remarked that division involves classification and classification division. Miss Jones therefore is right when she remarks, "Division and classification are the same thing looked at from different points of view," and a table of division is at once a table of classification. But in spite of this there is a distinction between the two. According to Mill the doctrine of division gives place to that of classification when we adopt a material standpoint. In division, as we have found, we cannot altogether ignore facts, yet it is more or less formal and limited in investigating facts. But classification is impossible without putting material considerations first and foremost. Thus classification, more than division, helps towards the progress of knowledge. In division we pass from unity to multiplicity, in classification from multiplicity to unity, or at least to a system. Joseph distinguishes between division and classification by stating that though division is closely allied to classification and definition, "the difference between division and classification seems to be principally this, that we divide the genus, but classify the particulars belonging to it. In other words, division moves downwards from the more general to the more special, classification upwards from the particulars through the more special to the more general." The problem of classification, according to Mill, is that "classification

An explanation of the nature of classification and a comparison between division and classification.

should be to provide that things shall be thought of in such groups, and those groups in such an order, as will best conduce to the remembrance and to the ascertainment of their laws." Welton explains the nature of classification thus :—"A......development of logical division on the material side leads to the theory of classification. The object of classifying is to so arrange in order the facts with which we are dealing that we can the most easily acquire the greatest possible command over them, and can economise statement—and so lighten the task imposed on memory—by being enabled to convey a large amount of information in a few words."

Names and classification.

General names such as man, house, tree etc. classify things for us. Predicables and categories, though names of classes, are not the names of natural classes according to Mill. Mill says that there are general names which are not class names, *e.g.* god, mermaid, ghost, etc., but there are other general names which are really names of classes, such as animal, plant, dog, elephant etc. "There is a classification of things which is inseparable from the fact of giving them general names" (Mill). Every connotative name divides things into two classes, *viz*. those which have this connotation and those which do not have this connotation. We are here concerned not with how names classify things, but with the grouping of things, which are afterwards given general names as a consequence of classification.

Following the Scholastic logicians Mill draws a distinction between **natural** classification and **artificial**

The distinction between natural and artificial classification.

classification. Natural classification is the grouping of things according to important and numerous points of similarity, while artificial classification is the grouping of things according to some purpose in view. Thus it is supposed that Nature has herself arranged things in classes, and to group things according to Nature's plan is scientific, as in the grouping of animal species according to the degree of perfection of animal life. On the other hand artificial classification is made according to the purpose of the individual concerned, as in grouping words in a dictionary or grouping books in a library catalogue in alphabetical order. In such a case the members of a group have no inner affinity. The words 'man' and 'mountain' are grouped together in a dictionary, though the things denoted by the terms have no affinity. But the grouping of books in a library in different book-cases according to subject-matter, *e.g.* books on logic, history, physics and so on, is scientific, inasmuch as there is a natural affinity between the members of every group. According to Mill the grouping of plants with reference to stamens and pistils is not scientific and natural classification, but artificial. Similarly Bentham's classification of flowering plants is artificial. He classifies British flora into those whose flowers are compound and those whose flowers are not compound, and subdivides the former into those with one seed and those with more than one seed, and so on. Again the classification of plants into monocotyledons and dicotyledons is, according to Mill, artificial.

It may be pointed out that natural classification is

objective, because in this case things are classified according to their resemblance or affinity, while artificial classification is subjective, inasmuch as in this case the interest of the individual, that is, the purpose he has in view, determines classification. Thus the classification of plants by a medical man is not the same as the classification of them by a farmer or a botanist.

Criticism of the distinction between natural and artificial classification.

Natural classes were supposed by the Schoolmen and others to be fixed and unalterable. But the science of biology has shown that classes cannot be shut up in watertight compartments. Various species and genera can be traced to a common stock and there is a family relation between them. Besides, in the course of evolution a particular class may undergo metamorphosis, and change its character altogether. So classification should be effected not according to resemblance but according to family relationship. Modern books, therefore, set out family relationships by means of 'genealogical trees'. Further, according to Welton there is no essential difference between artificial and natural classification. Every classification is artificial and is based upon some purpose, yet it aims at being natural. We classify according to some idea. He distinguishes however between classification for some special purpose, analogous to artificial classification, and classification for a general purpose, similar to natural classification. Besides, we may remark that artificial classification is not useless, but has proved very valuable in many cases. Instead of speaking of natural and artificial classification

we should rather distinguish between **Scientific** and **popular** classification. Botanists and zoologists have attempted to give us scientific classifications of plants and animals, and their attempts have proved very valuable for the progress of knowledge. We should also remark that classification should not be made once for all. Old classifications must give way to new ones with the advance of knowledge. The old classification of plants into trees, shrubs and herbs has been found to be unscientific.

In the light of what has been said above we may provide some rules of classification. The rules of division given previously hold good for classification as well. The classes should be exclusive, and the sub-classes together should be equal in extent to the genus divided. In classifying, a single principle should be observed, etc. Besides these, we may note certain special rules of classification, *viz.*—(1) "The classification should be appropriate to the purpose in hand" (Welton). (2) The higher the group the more important should be the attributes by which it is constituted. (3) The classification should be graduated, so that the groups with most affinity with each other may be nearest together, and so that the distance of one group from another may be an indication of the degree of their dissimilarity. (4) All groups should be so constituted as to differ from each other by a multitude of attributes. These rules are difficult to satisfy. How are we to find out the most important attributes? Our knowledge is not perfect. To arrange classes in a gradual order must depend upon knowledge of the

The rules of classification.

affinity between different classes. It is very difficult to find out this affinity.

According to Whewell, classification should be according to **type** and not according to definition. Species are not clearly marked off and one runs into another. Natural history provides us with classification according to type. Such classification, according to Mill, is unnatural and therefore unscientific, while mathematics and physics give us scientific classification. A type is an example of a class which embodies in a prominent degree the leading and important characteristics of the class. Thus in classifying monkeys according to type we may take one typical example of the class, which has the leading characteristics of the class. But classification is not determined from without but is determined by a central point within, that is, by internal affinity and not by external marks. According to Jevons, the type is an individual and no other individual is like it. If a type is found out by the selection of a few important qualities, this can only be the result of a knowledge of classification, that is, by a knowledge of the connotation of the class name. Thus classification by type is circular, since to have the type we require a knowledge of the general attributes of a class. Thus classification by type cannot be defended logically. Besides, we classify things not according to denotation but according to connotation. We can no more classify by type than we can define by it. Though classification by type has been found to be unscientific, it is not useless. The older logicians attempted

An explanation of classification by type and some criticisms of it.

such classification, and types often help us to find out class properties.

Comte was the first to recognise classification according to **series**. Classes are marked by mutual affinity, and if they are serially arranged, it becomes easy to find out very general laws. Different animal classes are the modifications of animal life. 'How to arrange the classes side by side, not how to arrange individuals, is the problem of classification by series. One species often passes into another, and this again into a third, and so on. Thus to arrange groups according to nearness of relation is serial arrangement.' An ellipse passes into a circle when its diameters become equal. It passes into a straight line when the conjugate diameter becomes *nil*. An ellipse is thus intermediate between a circle and a straight line. According to Welton the arrangement of natural classes is not serial. It rather resembles an arrangement of concentric circles in a globe. Classification in zoology is according to the degree of perfection of animal life. A species is perfect, not when it is between two extremes, but when it passes to some other species. Thus we do not have a series but a graduated order in natural classification. Man is not intermediate between animal and dog, but in relation to animal both man and dog have the same position.

An explanation of classification by series.

Scientific classification, aided by scientific nomenclature, contributes largely to the advancement of knowledge. The things of the world are innumerable and varied. To group them under classes is a great aid to memory.

The uses of classification.

Without classification they would become unmanageable. Classification helps us to find out general laws, without which knowledge cannot progress. Again if classes are arranged in graduated order according to affinity and nearness, we can compare conveniently and draw inferences by analogy. Thus zoology and botany give us genealogical trees. Thus we find that scientific classification is indispensable for the advancement of science.

Nomenclature and Terminology

Names invented by science are well defined and have their meanings fixed; *e.g.* the terms point, triangle etc. in geometry. But the meanings of names popularly used vary from time to time according to usage and custom. Thus we find in the dictionary that a word has different shades of meaning. The word pagan originally meant a villager, but now it means a heathen. Fidelity meant faithfulness to the oath of allegiance, but now it means any kind of faithfulness. These examples show how words undergo a change in their meaning. Words change their meaning either by **generalisation** or by **specialisation**. When words are generalised their connotation diminishes. Oil, which originally meant olive oil, now means any kind of oil. This is an example to show how a word changes its meaning by generalisation. Psychologists and sociologists often generalise words. When words change their meaning by specialisation, their connotation increases. Logicians should accept words in general use, but should define them clearly and precisely for the fulfilment of their purpose.

Words in general use change their meanings; scientists when they use them ought clearly to define their meaning or else invent technical terms.

The problem of nomenclature explained.

Nomenclature is a system of names for classes. According to Mill, "A nomenclature may be defined as the collection of the names of all the kinds with which any branch of knowledge is conversant; or more properly, of all the lowest kinds, or infimae species—those which may be sub-divided indeed, but not into kinds, and which generally accord with what in natural history are termed simply species." No classification can remain fixed without a corresponding nomenclature, and every good nomenclature involves a good system of classification. Whewell says, "System and nomenclature are each essential to the other." If classes lack nomenclature, the progress of thought becomes impossible. Names of classes have not only denotation, but they have also connotation conventionally fixed. Though artificial classification may have nomenclature, such nomenclature is not scientific, since the same thing may be artificially grouped under different classes. Botany, zoology and chemistry give us excellent systems of names or nomenclatures for classes. In botany higher classes have been given names such as Dicotyledon, Rosa, Geranium etc. "The species is marked by adding a distinctive attribute to the name of the genus, as viola odorata, orchis maculata etc." (Welton and Monahan). These distinctive attributes do not stand for differentiae. Sometimes a class is named after some individual, *e.g.* Rosa Wilsoni; sometimes after some country, *e.g.* Anemone Japonica; sometimes from some peculiarity of the plant, as Geranium Sanguineum; some names are fanciful, *e.g.* Bauhinia. We have from the time of Linnaeus the names of higher classes of

plants. Chemistry gives us another method of naming classes, based upon the oxygen theory. The principle here adopted as the basis of scientific nomenclature is founded on a modification of the relation of elements. We have thus "sulphuric and sulphurous acids, sulphates and sulphites of bases, and sulphurets of metals; and in like manner, phosphoric and phosphorous acids, phosphates, phosphites, phosphurets" (Welton and Monahan). Such naming at once implies the place of a thing in a system. The three oxides of iron are protoxide, the black oxide and the peroxide.

The problem of Terminology explained.

We should not only have scientific names for classes, but we should also have scientific terms to be able adequately to describe individual things. Such a system of scientific terms necessary for the description of individual things is called **Terminology.** Thus while nomenclature is a system of names for classes, terminology is a system of names for parts or qualities of individual things. Botany provides us with examples of such a terminology. Heré also we are indebted to Linnaeus. Thus the parts of flowers have been distinguished as calyx, corolla, stamens and pistils. Names of the parts of plants are pistil, stamen, calyx, frond, and names of properties are bipartite, silicate, pinnate. When we use current names, we should fix their meaning conventionally, and technical names should be well defined and taught through the knowledge of objects. The knowledge of the colour implied by a name can only be taught through the eye.

BOOK II

PROPOSITION

CHAPTER I

The Definition and Nature of Proposition

The problem of **proposition** in logic is so central and important that it is essential to treat it with some fulness, though the subject will appear to beginners difficult to grasp. Logic is the science of thought; we think only when we judge; and there is a close connection, if not identity, between judgment and proposition. The judgment is the unit of thought in the same way as the sentence is the unit of language. Ignoring the distinction between judgment and proposition for the present, and regarding them as identical, we may state with Johnson that "a systematic treatment of logic must begin by regarding the proposition as the unit from which the whole body of logical principles may be developed." According to him, "A proposition is that of which truth and falsity can be significantly predicated." Russell also regards proposition as anything which is either true or false.

Definition of Proposition.

Only assertions, that is, affirmations or denials, can claim truth, and we cannot assert without judging. So judgment and proposition are closely related. What then is the relation between them? According to Bosanquet, "Judgment claims to be true, that is,

The relation between judgment and proposition.

presupposes the distinction between truth and falsity." Thus he defines judgment in the same terms in which Johnson and Russell define proposition. Should judgment and proposition then be regarded as identical? Johnson argues that this is not possible, because judgment is concerned with the mental attitude of the person judging and is subjective, or as he says, *epistemic*. He does not agree with Bosanquet and others, who hold that a proposition is the expression of a judgment in words or language, because a mere verbal expression cannot claim truth; the expression of a judgment in language is a sentence, not a proposition. So according to him judgment is wider than proposition. Every proposition, however, involves judgment, since there cannot be any assertion without judgment. Therefore he says that we pass judgment upon propositions, which stand for some fact, that is, are objective or *constitutive* in nature. Anything that is true or false must have objective reference and must also imply assertion. But, we may ask, if proposition is regarded merely as objective, as it is by Johnson, how can it claim truth? If however it claims truth, it must combine both the subjective and the objective aspects of knowledge, and therefore judgment and proposition must be inseparable. This means that a proposition is at the same time a judgment. Without fulfilling this condition proposition cannot claim truth, and cannot form the central problem of logic, from which all other logical principles can be derived.

Logicians have generally used the terms judgment and proposition in the same sense, and we can avoid confusion

Identity between judgment and proposition.

if we can find some clue to identify them, or at least to find a very close relation between them. (One school of logicians has treated of judgments exclusively without any reference to propositions, another has treated of propositions alone without any reference to judgments. But this has led to difficulties. The former school ignores the close connection between thought and language, while the latter concerns itself mainly with the grammatical structure of proposition and its different forms. It appears to us that a compromise can be effected between Bosanquet and Johnson, since the former's definition of judgment is the same as the latter's definition of proposition. If we do not take judgment in its wide and psychological sense, but narrow its meaning for logical purposes, we can rightly say, as Bosanquet does, that **judgment** claims to be true, *i.e.* **presupposes** the **distinction between truth and falsity.** What then is a **proposition** ? It should not be defined as the expression of a judgment in language, but rather as a **judgment expressed in language.** If we accept this definition, we can at once say that both judgment and proposition claim truth, the only difference being that the proposition has linguistic reference, while judgment has not. We may also say that in logic we are concerned with propositions as understood, and a judgment is nothing but a proposition as understood. Thus instead of regarding judgment as merely subjective and proposition as merely objective, we regard both judgment and proposition as having both subjective and objective aspects, or as Johnson puts it, epistemic and constitutive aspects.) We can reduce logic neither to psychology and

metaphysics nor to grammar. If we decide to regard judgment and proposition as essentially the same, it will be possible for us to use the terms judgment and proposition in the same sense indifferently.

What do we mean when we say that a judgment or proposition claims truth? We have already remarked

Subjective and objective aspects of judgment or proposition.

that every proposition has two aspects, subjective and objective. Whenever we judge, we judge about something, and this something to which every judgment is referred is its objective aspect. Every judgment has a subjective aspect as well, because judgment is a mental act, and involves belief or disbelief. This mental attitude is present in every judgment or proposition. To assert something is a mental process, and assertion is the essential mark of judgment. Consider the judgment 'gold is yellow.' It is a mental act, and I cannot say that gold is yellow without believing in the proposition. But if it be merely subjective, it cannot be either true or false. A mental state by itself, whether sensation, feeling or volition, cannot be regarded as either true or false. My idea of gold being yellow, if it is to be true, must have reference to reality or fact. That is, every judgment in order to be true must correspond with some portion of reality. We may judge about the whole of reality or about some portion of it. According to Bradley, a judgment, in order to be true, must be compatible with some content of reality. Bosanquet also holds almost the same view, and according to him a judgment may have reference to some cross-section of reality or to the whole of it. The same thing is meant by the expression

'universe of discourse.' We refer every judgment to some universe of discourse, whether it be the real world or the world of fiction. The universe of discourse or the objective reference of the judgments 'gold is yellow' or 'all men are mortal' is the physical universe. The judgment 'fairies can assume different forms' refers to the universe of folklore. The universe of discourse of the judgment 'Hamlet killed Polonius' is the world of fiction. Thus we find that though every proposition is a mental act and is a continuous idea, yet since it claims truth, it has always reference to some aspect of reality or to some universe of discourse.

We must here make a distinction between the **metaphysical, grammatical and logical subject** of proposition.

Metaphysical subject of proposition, distinct from logical and grammatical subject.

In treating of terms we noted that every proposition when fully developed has two terms, *viz.* subject and predicate. In the proposition 'man is mortal' the grammatical subject is 'man.' But this proposition may be the answer to either of the two questions, 'what is mortal ?' and 'what is man ?' In the first case the **logical** subject is 'mortal,' because we enquire about it, and in the second case 'man' is the **logical** subject, though the **grammatical** subject of the proposition is always the same. But what is its metaphysical subject ? The **metaphysical** subject of the judgment stated above, which lurks behind the grammatical subject, is the physical universe in which 'man-being-mortal' is true. According to Aristotle, the ultimate or metaphysical subject of every judgment is some individual, and there is a plurality of such individuals.

According to Bradley, it is always one system of reality having a variety of contents. According to Bosanquet, the metaphysical subject may be either the whole of reality or a portion of it. According to Johnson, every judgment is universal and is of the nature of an adjective, while the ultimate subject is always particular, and is of the nature of a substantive. So since judgment corresponds with fact we may say with Johnson, following Aristotle, that the universal is in the particular. All these thinkers agree in holding that in every proposition there is a real subject behind the grammatical subject. Thus in logic we cannot dispense with metaphysics altogether. But though reference to reality is required by every judgment, we need not in logic enquire of the nature of reality. From what we have remarked we may conclude that **every judgment** is in a sense **existential**, that is, refers to some object real or fictitious. Even mythology and fiction have their place in the system of reality, and when judgments are referred to them, they cannot from this point of view be regarded as other than real.

Judgment is universal, necessary and constructive.

Every **judgment** has **three main characteristics,** *viz.* that it is universal, necessary and constructive. Judgment is **universal** because what is true is always true and what is false is always false. If the judgment 'man is mortal' be true, it is true to me, to you and to every other person, and it was true in the past, is true now and will be true in the future. If this is so, we cannot say that a true judgment may become false after a lapse of time. Some judgments have reference to parti-

cular points of time. Can such judgments be regarded as always true? Let us take the judgment 'Socrates is drinking hemlock.' Socrates is not drinking hemlock now, nor could he drink it before he was born. So the judgment has reference to a particular point of time. Can it then be regarded as always true? Our view is that it can, because in so far as the judgment has reference to a particular point of time it is always true. The fact that on a particular day and at a particular time Socrates drank hemlock cannot be denied. Similarly the judgment 'this rose is white' is always true, though the rose may change its colour with the passage of time, since the judgment refers to some particular point of time, and according to that reference and under that condition it is always true. Bosanquet, to make the matter clear, distinguishes between the time **of** predication and time **in** predication. The time of predication is the time when an individual judges, and time in predication is the time to which the judgment refers. If all this is true, then we cannot agree with Bosanquet that narrative judgments are not truly judgments since they require tense, as in the example 'Caesar crossed the Rubicon.' This judgment is as universal as the judgment 'all material bodies gravitate' in so far as the limitation of its reference to a particular point of time is considered.

Secondly, judgment is **necessary.** This means that in judging there is always some restraint imposed upon us by the system of reality or fact. We cannot judge as we like. We cannot say that men are quadrupeds, because the real world does not provide us with examples

of men with four feet. In judging we have always to take the actual world into consideration, if our judgment is to be true. The necessity of judgment, therefore, consists in the restraint imposed upon our thought by facts when we judge.

Lastly, judgments are **constructive.** Reality, as Bosanquet says, is a continuous judgment embracing the whole universe. The world to us is nothing but what we know of it. We make our world and construct it as we pass from judgment to judgment. Our world is the world of our judging mind, and our knowledge being progressive we construct the real world progressively. If there be anything outside our knowledge, it is nothing to us. So judgments are constructive, because they determine for us what our real world is.

Judgment not always expressed in propositions.

We have defined proposition as judgment expressed in language, and have pointed out that since thought is the subject matter of logic, judgment and proposition are identical in essence. But should every judgment be expressed in language? This is not necessary, though every judgment *may* be so expressed with more or less difficulty. Where pictorial thinking is possible we can judge without the help of language, as in chess-playing. But we require language to think of complex matters such as the British constitution, the government of India, the respiration of plants, the internal structure of animals, etc. In such cases we can hardly judge without the help of language. Though judgment and proposition are essentially identical,

traditional logic gives us forms of proposition some of which hardly correspond with judgments, and in formal logic we cannot but consider these different forms of proposition. We shall discuss them in the next chapter.

Nature and function of the copula in proposition.

In discussing terms we pointed out that every proposition must have **subject** and **predicate** when fully developed. The judgment 'plants are living' has for its subject 'plants' and for its predicate 'living.' Here however we are considering the grammatical structure of a proposition, and speak of the grammatical subject and predicate. The above proposition includes another word, 'is,' which is called the **copula** of the proposition. It is commonly held that the copula should always be some form of the verb 'to be.' But this is not necessary, since in inflection we may have a proposition in which 'is' is absent, *e. g.* 'fire burns.' Here the copula and the predicate are not two words, but are both present in the one word 'burns.' So we may have propositions in which the copula as a form of the verb 'to be' is absent.

What then is the **function** of the **copula**? According to Mill it is merely the **sign of predication.** 'Is' implies affirmation and 'is not' denial. In the proposition 'matter is extended' the predicate 'extended' is affirmed of matter, and the function of affirmation is performed by the copula 'is'. In the judgment 'some men are not happy' the predicate 'happy' is denied of the subject 'some men,' the copula 'is not' performing the function of denial. So the copula should **not** be regarded as a **third term** in a proposition. It is, according to Bosanquet, "the grip

with which the parts of a single complex whole cohere with one another, differing according to the nature of the whole and the interdependence of its parts." According to Johnson, it is the tie that connects the subject and the predicate into a single whole. We have already remarked that every proposition is a continuous idea and is a single act. The copula implies the act of judgment. Though from the grammatical point of view the copula appears to be a third member of a proposition, it is not so, if we consider the significance of the proposition. Besides, we shall subsequently find that there may be propositions in which both the grammatical subject and the copula are absent. Predication is the main function of proposition.

Another point has to be cleared up. Does the copula 'is' imply existence? It does not. In the proposition 'man is mortal,' it is held that 'is' is not merely a sign of predication but implies existence as well. This is not true. We judge about fictitious objects; *e. g.* 'the centaur is a product of the poetic imagination.' Here 'is' does not imply real existence. Though every proposition may be regarded as existential in some sense, strictly speaking the only existential judgments are those in which the term 'exist' occurs, *e.g.* 'matter exists.'

A distinction between **proposition** and **sentence** may be drawn here. The sentence is the unit of language, the proposition is the unit of thought. A sentence is either correct or incorrect, while a proposition is either true or false. Though every proposition is a sentence, every sentence is not a proposition. Thus optatives, *e.g.*

Sentence and proposition.

'Let the king live long,' imperatives, *e.g.* 'Go home !' and questions, *e.g.* 'Are you coming ?' are sentences but not propositions, as they do not claim truth. We have said that in a developed proposition both the grammatical subject and the copula are present, but there are judgments in which either the subject or the copula or both are absent. Exclamation may be regarded as the most elementary form of judgment, in which both the subject and the copula are absent. After looking at a thing we may say, 'Beautiful !' This exclamation is a judgment meaning that 'that is beautiful.' But in the proposition 'Beautiful !' both the subject and the copula are absent. Similarly 'Lightning !' is a judgment implying 'there is lightning' or 'that is lightning.' In impersonal propositions, *e. g.* 'it is raining,' 'there is a British constitution that protects the liberty of the people,' etc., the whole proposition is concentrated in the predicate, and practically speaking no grammatical subject is present. In such judgments as 'that is a house,' 'this is a book,' etc., which may be called demonstrative judgments, though the grammatical subject is present it is indefinite. Thus gradually there is a process of development of propositions, and we pass from exclamatory propositions to impersonal propositions, and from these to demonstrative propositions, till we reach propositions in which the subject is definite and the parts of the proposition are fully stated, *e. g.* 'Socrates is mortal,' 'man is mortal.' In the next chapter we shall discuss the forms of proposition, beginning with the traditional scheme.

CHAPTER II

FORMS OF PROPOSITIONS

Various **classifications of propositions** have been offered by logicians, no one of which can be regarded as complete by itself. The traditional scheme of classification, which is simple though incomplete, gives us a provisional basis for discussing the forms of propositions.

Traditional and Kantian classifications.

Traditional logic classifies propositions according to **quality** into **affirmative** and **negative**; according to **quantity** into **universal** and **particular**; and according to **relation** into **categorical** and **conditional**, the last being further subdivided into **hypothetical** and **disjunctive.** **Kant** gives us a fourfold classification of propositions. According to him propositions are, in respect of quality, affirmative, negative and **infinite**; in respect of relation, categorical, hypothetical and disjunctive; and in respect of **modality, apodeictic** or necessary, **assertoric** and **problematic.**

We shall provide here a scheme of classification which is accepted in a general way by modern logicians, though our basis of discussion will be the traditional scheme of classification. We shall in the next chapter briefly discuss the Hamiltonian scheme of classification by quantifying predicates, and the equational theory of Jevons, which also involves the quantification of predicates. Though Bosanquet has thrown much light

upon the problem of the forms of proposition, and we shall have to improve the traditional scheme in the light of his discussion, yet his classification showing the development of judgments, though useful, is rather psychological than logical, and it is not possible to accept his scheme.

In the last chapter we showed that in certain judgments only the predicate is present, neither the subject nor the copula being expressed, and that in an impersonal judgment the implication of the judgment is to be found entirely in the predicate. But even such judgments can be stated fully in a form in which the subject, the copula and the predicate are all present. Thus the judgment 'house !' may be translated as 'that is a house.' Similarly the judgment 'it rains !' may be developed into the proposition 'rain is falling.' So a judgment when fully developed presents us with subject, predicate and copula, and every judgment may with more or less difficulty be fully expressed. We may further remark that though proposition is a single act of assertion, yet the analysis of its linguistic form gives us the grammatical subject and the grammatical predicate, behind which the real subject is always present. In discussing the forms of proposition we shall have to bring out the implications of the grammatical subject and predicate, and of the relation existing between them.

The Fourfold Scheme of Classification

Traditional logic gives us a fourfold classification of propositions. According to **quality** propositions, as we

have seen, are either **affirmative** or **negative.** An **affirmative** proposition **affirms a predicate of a subject,** *e.g.* 'cows are domestic animals' or 'John is an Englishman.' In a **negative** proposition the **predicate is denied of the subject,** *e. g.* 'men are not perfect,' 'Socrates is not a Roman.' We may here note that in a negative proposition the copula is 'is not,' which is the sign of denial. Hobbes attempts to regard all propositions as affirmative by supposing that the negative sign 'not' is a part of the predicate. So according to him in the proposition 'men are not perfect,' 'not-perfect' is affirmed of the subject 'men.' But this is a misreading of the meaning of the proposition. In this proposition what we are really doing is to deny the predicate of the subject, and this denial is performed by the copula 'are not.'

Affirmative and negative propositions.

One other point may be noted here. What is the meaning of denial? When we say 'snow is not hot,' it is because we know it is cold. Absolute denial is impossible in thought. Negation has always a positive background, though this is not explicit. Snow is not hot, because it is cold; this book is not large, because it is small; this flower is not white, because it is red. Thus we deny to affirm. Truth often comes home to us through denial, because if we can find out what a thing is not, we can determine what it is. So according to Plato negation is as useful as affirmation. The function of negation will be

Negation has an implied positive background.

more clearly understood when we discuss induction, since without negation, that is, elimination of irrelevant circumstances, inductive generalisation is impossible. Discrimination and differentiation are important elements of thought which are effected by negation.

Distinction between negative and infinite propositions not tenable.

Besides affirmative and negative propositions, Kant gives us another distinction according to quality, *viz.* **infinite** propositions. The form of an affirmative proposition is 'S is P,' that of a negative proposition 'S is not P,' while the form of an infinite proposition is 'S is not-P.' Thus the proposition 'this flower is not-red' is an infinite proposition. But what does this proposition really mean? Is an infinite proposition distinct from a negative proposition? This cannot be established. 'This flower is not-red' means that this flower excludes red, and this can be expressed by the negative proposition 'this flower is not red.' If 'not-red' means nothing at all, the proposition becomes nonsense. If it means anything, it must mean 'something other than red.' If so, then the negative proposition 'this flower is not red' brings out the meaning quite clearly. So to distinguish infinite from negative propositions not merely introduces tautology but may become misleading. Later however we shall find that for convenience formal logic often takes recourse to the form not-S, not-P etc. One other point may be mentioned. Logicians have regarded infinite propositions in the form S is not-P as positive, while the proposition in the form S is not P has been regarded as negative. This distinction, though formally allowable,

cannot be regarded as valid if the meaning of such propositions is taken into account.

According to **quantity** propositions are distinguished as **universal** and **particular.** A **universal** proposition is one in which the predicate is affirmed or denied of the **entire denotation of the subject**, *e. g.* All or every S is P, all triangles are rectilineal figures, no men are perfect, etc. A **particular** proposition is one in which the predicate is affirmed or denied of a **part of the denotation of the subject**, *e. g.* Some S's are *or* are not P, some men are not happy, some flowers are red, etc. Certain **marks of quantity** are recognised by logicians. All, every, no, each, any, etc. are marks of universal propositions, while some, a few, not all, most, etc. are marks of particular propositions. A categorical proposition, which is one in which the predicate is affirmed unconditionally of the subject, has according to traditional logic four elements, *viz*. the subject, the predicate, the copula, and the mark of quantity. Thus in the proposition All S is P, S is the subject, P the predicate, 'is' the copula and 'all' the mark of quantity.

Universal and particular propositions.

Now combining the distinctions according to quality and quantity we get the **fourfold scheme** of classification, *viz*. Universal affirmative (A)—'All S is P;' Universal negative (E)—'No S is P;' Particular affirmative (I)—'some S is P;' Particular negative (O)—'Some S is not P.' Thus the **symbols** A, E, I and O stand respectively for universal affirmative, universal negative,

The fourfold classification of propositions.

particular affirmative and particular negative. For convenience we shall follow several other logicians and use in this book the symbols S a P for universal affirmative, S e P for universal negative, S i P for particular affirmative, and S o P for particular negative propositions. These symbols not only give us the quality and quantity of a proposition but also indicate the subject and the predicate. Thus the symbol S a P gives us S as the subject, P as the predicate, and the sign a tells us that the proposition is universal and affirmative. Thus S a P is equal to All S is P; S e P = No S is P; S i P = Some S is P; S o P = Some S is not P. Further, we shall use such symbols as S′ for not-S, P′ for not-P, Q′ for not-Q and so on. Thus the proposition S′ a P′ = All not-S is not-P, S′ e P = No not-S is P, S i P′ = Some S is not-P, and so on.

Distribution of terms.

A **term** is said to be **distributed** when **it** is **taken in its entire extent or denotation.** A **universal** proposition **distributes** its **subject** term, while a **negative** proposition **distributes** its **predicate** term. Thus universal affirmative propositions (A) distribute their subjects only, while universal negative propositions (E) distribute both their subjects and their predicates. Particular affirmative propositions (I) distribute neither the subject nor the predicate, while particular negative propositions (O) distribute their predicates only and not their subjects. Thus affirmative propositions do not ordinarily distribute their predicates, and particular propositions do not usually distribute their subjects. The universal affirmative proposition All S is P = All S is some P; the

universal negative proposition No S is P = No S is any P; the particular affirmative proposition Some S is P = Some S is some P; the particular negative proposition Some S is not P = Some S is not any P. When we say that all men are mortal, we mean that some things denoted by the term 'mortal' are identical with all things denoted by the term 'man.' Similarly the proposition 'no men are perfect' means that all things denoted by the term 'perfect' are outside all the things denoted by the term 'man.' In the same way the proposition 'some men are virtuous' means that some things denoted by the term 'virtuous' are identical with some things denoted by the term 'man.' The proposition 'some flowers are not red' implies that all things denoted by the term 'red' are other than some things denoted by the term 'flower.'

We may note that the theory of distribution of terms in a proposition rests upon the assumption that the subject and the predicate of all propositions are read in their denotation or extension and not in their connotation or intension. We shall subsequently find that the denotative view of predication is not satisfactory.

To sum up the theory of distribution, we may point out that if only one term of an affirmative proposition is distributed, it is the subject term, and if only one term of a negative proposition is distributed, it is the predicate term. But an affirmative proposition may in some cases distribute its predicate as well, as in the case 'all equilateral triangles are equiangular.' (This is because the subject and predicate are here co-extensive.)

A universal negative proposition also distributes both its subject and its predicate terms ; *e.g.* 'No men are four-footed.'

Hitherto we have discussed universal and particular propositions in a general way, but it is necessary now to consider them with greater fulness. (**Universal** propositions may be either **singular** or **general. Singular** propositions are those in which the predicate is affirmed or denied of an **individual subject,** *e.g.* 'Alexander is the son of Philip,' 'London is the largest city in Europe,' 'my boat' is on the shore,' 'Napoleon is not an Englishman,' etc. Such propositions are universal because in them the predicate is affirmed or denied of the entire extension of the subject. Singular propositions may be indefinite, and from the logical point of view are then equivalent to particular propositions, *e.g.* 'a certain man is ill,' 'a ship is sailing.' **Collective** universals are equivalent to singular universals, *e.g.* 'All the angles of a triangle are equal to two right angles,' 'all the books in the library weigh several tons,' 'the Romans conquered Gaul' (the reference being to the Roman army as a body), etc.

Different forms of universal propositions : 1. Singular propositions.

In a singular universal proposition the subject is logically indivisible, while in a general universal the subject is undivided, *e.g.* 'all men are mortal'. (**General** universals again may be either **numerical** or **empirical generalisations** or **abstract universals.**) In the propositions 'all my sons are in the army' or 'all the

2. General universal propositions.

students of this class are expected to pass the next examination', etc., we have examples of **numerical** universal propositions. Such propositions as 'all lions are tawny', 'all scarlet flowers are without sweet scent' etc. are examples of **empirical** universals, that is, of laws based upon experience. Numerical universals and empirical universals, like singular universals, are **concrete,** as they directly refer to facts of experience. But **abstract** universals assert a **relation of content** and are not concrete, *e.g.* 'every right-angled triangle can be inscribed in a semi-circle', 'all triangles have their angles equal to two right angles', 'all men are mortal', etc. Abstract universals, since they assert a relation of content, are analogous to hypothetical propositions; indeed they mark a transition from the categorical to the hypothetical form of propositions. 'All' is a sign of a universal proposition, but it may be either distributive, as in 'all men are mortal', or collective, as in 'all the books in the library weigh several tons'. 'All' therefore is ambiguous. To get rid of this difficulty we may use the form All S's are P or Every S is P when it is distributive, and the form All S is P when it is collective.

The nature of particular propositions.

While the application or **denotation of the subject** term is definite in universal propositions, it is **indefinite** in the case of **particular** propositions, *e.g.*, 'Some flowers are red', 'some books are interesting', 'some men are **not** happy', etc. 'Some' is **not** inconsistent with 'all'. When we say that some men are intelligent, we do not exclude the possibility of all men's being so. 'Some' *is* inconsistent with 'none', and must

imply at least one. According to another view which is accepted by many, 'some' excludes both 'none' and 'all'. It appears to us that the first view is more satisfactory, because we use a particular proposition when our knowledge is imperfect; when we have learned more, we may find that we can use a universal proposition in its place. When we say that some lions are tawny, we do not exclude the possibility of all lions being tawny. Particular propositions do not assert a necessary relation of content. But this does not mean that they are useless, as Bosanquet holds, for we very often have to use particular propositions if we wish to give exact expression to what we are thinking. According to Keynes, particular propositions are important because they definitely deny something, and also because they assert existence. When we say that some flowers are red, we definitely deny the proposition that no flower is red. Again when we say that some engines can pull a train at the rate of 60 miles an hour for a long distance, we assert that such engines exist.

Indefinite or indesignate propositions.

Certain propositions are described as **indefinite,** though it is better to follow Hamilton in calling them **indesignate**, since we use the term 'indefinite' to indicate particular propositions. Indesignate propositions are those which lack the sign of quantity. Thus 'heat is a mode of motion,' 'bodies have weight', 'trains run at frequent intervals' etc. are indesignate propositions. The meaning or the context of such propositions shows whether they are universal or particular. Thus of the above propositions, the first two are universal,

while the last is particular. When we say that trains run at frequent intervals, we mean that *some* trains do so. When we say that bodies have weight, we mean that *all* bodies have weight.

Plurative propositions.

Such propositions as '**most** S's are P', 'most Indians are Hindus', '**few** S's are P', 'few men are geniuses', etc. are called **Plurative** propositions. Plurative propositions are particular because they are indefinite. 'Most' does not exclude 'all'. 'Most Indians are Hindus' means that 'some, more than half, Indians are Hindus.' 'Few' has a negative force. 'Few' is consistent with 'none' but excludes 'all'. 'Few men are geniuses' is equivalent to 'most men are not geniuses'. Therefore the proposition is an O proposition. Few S's are not P = Most S's are P, and is therefore an I proposition. Though plurative propositions are particular, one distinction between plurative and other particular propositions should be noted. From two plurative propositions a conclusion may be drawn. But from two particular propositions no conclusion follows. Thus from the propositions 'most S's are M', 'most S's are Q', we may deduce the conclusion 'some Q's are M'. But from the propositions 'some S's are M,' 'some S's are Q', we can draw no conclusion.

Numerically definite propositions.

Numerically definite propositions are those in which something is predicated of some definite proportion of a class, *e.g.* 'two-thirds of S's are P', 'sixty per cent of S's are P,' 'three-fourths of the members of the Bengal Council are intelligent men,' etc.

Plurative propositions and numerically definite propositions are **Exponibles.** Exponible propositions are those the full meaning of which can be brought out by two propositions. Thus 'most Europeans are Christians' means that 'some, more than half, Europeans are Christians' and also that 'few Europeans are not Christians'. Similarly when we say that 'two-thirds of S's are P' we at the same time mean that 'one-third of S's are not P'.

Exponible propositions.

An **Exceptive proposition** is one in which the quantity of the subject is limited by exceptions. If the exception is definite, the proposition is universal; if indefinite, it is particular. 'All S's except five are P' is a universal proposition, whereas 'all S's except a few (or some) are P' is a particular proposition.

Exceptive propositions.

In some propositions **predication** is **limited** by the consideration of time or some condition. In such cases we have examples of **multiple quantification,** *e.g.*, 'all men are sometimes unhappy', 'in some countries all foreigners are unpopular.' These are examples of secondary quantification. We may have examples of triple quantification as well, *e.g.* 'in all countries all foreigners are sometimes unpopular'. Such propositions may be either universal or particular, according to their meaning.

Multiple quantification.

Propositions are distinguished according as they are propositions **secundi adjacentis** or propositions **tertii**

Propositions secundi adjacentis and tertii adjacentis.

adjacentis. In the former the copula is not separated from the predicate, *e.g.* 'the man runs,' 'all that love virtue love angling.' In propositions tertii adjacentis the subject, the copula and the predicate are fully stated, *e.g.* 'no men are perfect,' 'all lovers of virtue are lovers of angling.'

Traditional logic requires every proposition to be stated in its logical form.

Traditional logic requires that every proposition should be stated in its **logical form**, so that it can be determined whether a given proposition is A or E or I or O. A logical proposition therefore should have a **subject**, a **predicate**, a **mark of quantity**, and also a **copula** which must always be some form of the verb 'to be' in the present tense. We often think in terms of propositions in which not all these elements are present. Thus the proposition 'None but the brave deserves the fair,' when cast into logical form, becomes either 'All that are brave are deserving of the fair,' or 'no non-brave persons are deserving of the fair'. Similarly, 'All that love virtue love angling'='all lovers of virtue are lovers of angling.' In the same way, 'the virtuous alone are happy' ='all that are virtuous are happy' or 'no non-virtuous persons are happy.' So also, 'Most men are guided by self-love' should be read as 'some, more than half, men are guided by self-love'. 'Few Indians are Christians'='some, more than half, Indians are not Christians'. 'Not all birds are black'='some birds are black' or 'some birds are not black'. 'Three-fourths of S's are P'='all S's excepting one-fourth are P.' 'All men are not happy'='some men

are happy and some men are not happy.' 'The man runs' ='the man is running.' 'Sweet are the uses of adversity' ='all the uses of adversity are sweet.' 'A few birds are musical'='some birds are musical'. 'All boys present excepting a few are well-behaved'='some boys present are well-behaved and some boys present are not well-behaved'. Many other instances might be cited.

Remarks on the traditional view of propositional forms.

Our discussion shows that forms of proposition do not always correspond to actual processes of thought. Our thoughts or judgments cannot always be translated into subject, predicate and copula. We have seen that there may be judgments in which both the subject and the copula are absent. To reduce judgments to propositional forms, therefore, sometimes involves distortion of thought. However, to satisfy the needs of formal logic we can with more or less difficulty throw every judgment into its logical form. Further, the distinction between subject and predicate is rather psychological than logical. The subject is that from which thought starts and the predicate develops it. The subject is not however in all propositions stated first; *e.g.* it comes last in 'ugly is the man.'

Distinction of Propositions according to Relation

According to **relation** propositions are either **categorical, hypothetical,** or **disjunctive.** Hypothetical and disjunctive forms are called **conditional.**

Categorical, hypothetical and disjunctive propositions defined.

A **categorical** proposition simply **affirms or denies a predicate of a subject,** that is, it makes an absolute statement; *e.g.*

'All S's are P', 'all virtuous men are trusted' or 'no birds are without wings'. "A **hypothetical** proposition is one in which the **predication** made in **one** proposition is asserted as a **consequence** from that expressed by **another**" (Welton). According to Joseph, "An hypothetical judgment **connects** a **consequent** with a **condition** which it does not however assert to be fulfilled." The forms of hypothetical judgments are—If P then Q; If S is P, M is N; If S is P, it is Q; or If S is P, P is Q. "A **disjunctive** judgment affirms **alternatives**" (Joseph), or asserts the truth of at least one of a number of alternatives; e.g. S is either P or Q; S is either P or Q or M.

Comparison of hypothetical and categorical judgment.

We have already explained the nature of a categorical proposition. We have shown that **individual** or **singular** propositions, including collective propositions, particular propositions, numerical propositions and empirical generalisations, are all **concrete,** because they directly refer to **facts,** while generic or **abstract universals** express **relations of content** and give us laws. Abstract universals therefore mark a transition between categorical and hypothetical judgments. **Hypothetical** judgments are **abstract, necessary** or **universal,** and they **assert a relation of content.** In hypothetical propositions direct reference to existing things is completely absent. Some abstract universals can easily be transformed into hypothetical propositions: 'Rainy weather is wet weather' is equivalent to 'if it rains, the weather is wet.' Every hypothetical judgment has an **antecedent,** called the protasis, which states the ground

or reason, and a **consequent**, called the apodosis, which makes the assertion.

Existence of antecedent not necessarily affirmed.

A hypothetical proposition does not invariably affirm the existence of its antecedent; *e.g.* 'If Hannibal had marched on Rome after Cannae he would have conquered it;' 'if a body is given a certain movement and if no counteracting conditions are operative, it will continue for ever to move in the same direction and with the same velocity.' In these examples we do not affirm the existence of the antecedent but only assert the relation of content, that is, we assert that if certain conditions are given certain events will necessarily take place. So in a hypothetical proposition, according to Cook Wilson, the solution of one problem depends upon the solution of another. Though this is true, hypothetical judgments, being judgments claiming truth, must have reference to reality or existence. The proposition 'if Hannibal had marched on Rome after Cannae he would have conquered it,' asserts that the condition of Rome was such that Hannibal would have conquered the city if he had marched upon it after Cannae, though he never did so.)

Scientific knowledge expressed by means of hypothetical propositions.

The progress of knowledge is from concrete judgment to abstract judgment, and our scientific knowledge, which is universal and necessary, is stated in the form of hypothetical propositions. Every hypothetical judgment is a single act of thought, though it expresses a relation between an antece-

dent and a consequent. Hypothetical propositions are called conditional because in them the assertion is limited by some condition, that is, involves a supposal.

Negative hypothetical propositions impossible, though formally recognised by logicians.

Can hypothetical propositions be negative? A hypothetical proposition is not negative simply because its antecedent or consequent is negative. Thus 'if S is not M, it is P,' 'if A is B, C is not D' are affirmative propositions. The view that whenever the consequent of a hypothetical proposition is negative, the proposition is negative, is untenable because even in such a case a relation between the antecedent and the consequent is asserted, *e.g.* If you daily walk for an hour in the morning you will not suffer from dyspepsia. (Indeed, the truth seems to be that a hypothetical proposition can never be negative, since its essential function is to assert the dependence of a certain conclusion upon a certain condition, and if any dependence is denied, we cease to have a hypothetical proposition. But by logicians the form If S is M, it is not P is sometimes, though less accurately, taken as a denial of the relation between the antecedent and the consequent, and regarded as a negative proposition.

Hypothetical propositions in their **denotative** form may become either universal or particular. When the antecedent has the universal sign, it is universal, and when it has the sign of a particular proposition, it is particular; *e. g.* If any S is P, it is M; sometimes if S is P, it is M; always if S is P, it is M; etc. A few concrete examples will illustrate the point; *e. g.* 'Some-

times when men are worried, they commit suicide'; 'always if a man is shot through the heart, he dies'; 'whenever a man moves, he expends energy'; 'sometimes if a man is ill, he cannot rise from his bed'; etc. Though hypothetical judgment is abstract and universal, instances may occur in which, though there is a connection between P and M, M may not be the full ground of P, or may not be universally operative, or may be counteracted by other influences. In such a case we have the form 'If S is M, it may be P,' or 'if S is M, it need not be P.' These are examples of **modal particulars.** Hypothetical judgments in their perfect form are ideal or abstract, universal and necessary, but if they are expressed in denotative form they become concrete and cease to be the perfect form of hypothetical judgments.

Distortion of meaning when hypothetical propositions are reduced to categorical ones or vice versa.

Abstract universals can easily be transformed into hypothetical propositions. Thus the proposition, 'Right-angled triangles have the square on the hypotenuse equal to the sum of the squares on the other two sides' = 'if a triangle is right-angled, the square on the hypotenuse is equal to the sum of the squares on the other two sides.' But in most cases a hypothetical proposition cannot be reduced to a categorical proposition, or vice versa, without distortion of meaning. Thus if the propositions 'gold is yellow,' 'man is mortal' etc. are translated into hypothetical form, we get the propositions, 'If gold is, it is yellow,' 'if man is, he is mortal,' which suggest the possibility of the non-existence of gold and man. Similarly the proposition, 'If men are honest, they do not deceive,'

may be translated into the categorical form 'All cases of men being honest are cases of men not deceiving.' Here the abstract meaning of the hypothetical proposition disappears when it is transformed into its categorical form and thus becomes concrete.)

We may now pass to the consideration of **Disjunctive Propositions.** 'We have noted that a disjunction asserts **alternatives.** These alternatives may be either two or more in number. Thus, 'S is either P or Q', 'S is either P or Q or R,' etc. are disjunctive propositions. **Disjunctions,** like hypothetical propositions, are **universal** and **necessary,** but every disjunction has a **categorical element** which hypothetical propositions lack.' Thus disjunctive propositions are more concrete than hypothetical propositions. In a **perfect** disjunction the **alternatives** are **exclusive** and **exhaustive,** *e.g.* 'This book is either historical or non-historical.' Here the subject must accept one of the alternatives, because they exclude each other and there cannot be any assertion outside them. Thus according to logicians such a proposition can be translated into four hypothetical propositions, *viz.* 'If the book is not historical, it is non-historical ; if it is not non-historical, it is historical ; if it is historical, it is not non-historical ; and if it is non-historical, it is not historical.') But we must observe that the disjunctive proposition stated above is not equivalent to the four hypotheticals taken together. Just as hypothetical propositions go beyond categorical ones, so also disjunctive propositions go

Nature of disjunction and its relation to hypothetical and categorical propositions.

beyond hypothetical ones. Further, every disjunctive proposition has a categorical element, and is intermediate between a purely categorical proposition and a purely hypothetical proposition.

There are some disjunctive propositions, however, which do **not** give us **exclusive and exhaustive** alternatives. Such disjunctive forms are not perfect. In the proposition 'This flower is either white or red,' we have alternatives which are **exclusive** but **not exhaustive**, since the flower may be neither white nor red but green. In this case, if we affirm one alternative, we can deny the other, but not vice versa. We can say, 'If this flower is white, it is not red, and if it is red, it is not white,' but we cannot say, 'If it is not white, it is red,' because it may be green. Such a form of disjunction is based upon the principle of contradiction, the form of which is 'S is not both P and Q.'

Examples of imperfect disjunction.

Again there are examples in which the alternatives are **exhaustive** but **not exclusive.** After seeing the striking successes of a certain man we may say that he is either intelligent or industrious. Here (in denying one of the alternatives we affirm the other, but if we affirm one we do not necessarily deny the other.) If the man is not intelligent, he is industrious, and if he is not industrious, he is intelligent; but it is possible that he may be both intelligent and industrious. So we cannot assert that if he is intelligent, he is not industrious. In such a case disjunction is based upon the principle of excluded middle, the form of which is 'S is either P or Q.' But if the alternatives be **neither** exclusive **nor** exhaustive, then there

is **no disjunction**, for then any assertion becomes impossible. Take the example, 'Every man is either honest or happy.' Here there is no disjunction, because a man may be neither honest nor happy, or he may be both honest and happy. So by denying one of the alternatives we cannot affirm the other, nor can we, by affirming one of the alternatives, deny the other.)

Disjunctions, like hypothetical propositions, may be **either universal or particular**. Thus, 'Some S's are either P or Q'; 'Some nations are either dependent or free'; 'Some men are either happy or unhappy,' are examples of disjunctive propositions which are **particular.** Similarly the proposition, 'Every idle man is either incapable of work or morally blameworthy,' is a **universal** disjunctive proposition. Disjunctives also resemble hypotheticals in that in their denotative form they cease to be universal and abstract, and become concrete. (Though disjunctions can be either universal or particular they must **always** be **affirmative.**) If negatived, they cease to be disjunctions and become compound propositions. 'S is neither P nor Q' is equivalent to the two propositions, 'S is not P and S is not Q.' To take a concrete example, 'This man is neither happy nor virtuous' does not tell us what the man is, and there cannot be any assertion in this case. It is a compound proposition.

Disjunctive propositions may be either universal or particular, but must always be affirmative.

Distinction of Propositions according to Modality

The **modal** distinctions of propositions involve very

Modal distinction of propositions according to Aristotle and Kant.

difficult considerations, and the problem may appear puzzling to beginners. Yet it appears desirable to provide a brief account at this stage.. When students have acquainted themselves with the principles of induction, they will find the problem easy of solution. **Aristotle** provides us with a **fourfold division** of modal propositions. They are (1) **Necessary**—'S must be P,' (2) **Contingent**—'S is P,' (3) **Possible**—'S may be P,' (4) **Impossible**—'S cannot be P'. Scholastic logicians regard necessary proposition as A, contingent proposition as I, possible as O and impossible as E. Aristotle's standpoint is objective and based upon the nature of the relation between the subject and the predicate. **Kant's threefold distinction** of propositions according to modality is generally accepted, but his view being subjective has rightly been criticised by different logicians. According to him a **necessary** proposition is universal and cannot even be reversed in thought. The form of such a proposition is 'S is P, necessary' (or 'apodeictic'). An **assertoric** judgment is simply accepted for the time being, but may be thought of as otherwise; it may be expressed as 'S is P, actual'. A **problematic** judgment expresses a doubt in asserting and may be expressed as 'S is P, possible.' According to Kant, necessary, assertoric and problematic judgments represent different degrees of belief in the mind of the person judging. According to Sigwart, to say that a judgment is necessary is not the same as to say that it is necessary for a predicate to belong to a subject. The former standpoint is Kantian, the latter Aristotelian.

Modal distinctions explained.

Since judgments claim truth, every true judgment is necessarily always true. Therefore Kant's subjective distinction between necessary and assertoric judgments cannot be accepted. A necessary judgment is an assertoric one, but the assertion is more emphatic. Both necessary and assertoric judgments imply complete belief, as every judgment claims truth. What then is the distinction between necessary, assertoric and problematic judgments? First we may point out that **necessary** and **problematic** judgments are **conditional**, while the **assertoric** judgment is **categorical.** When the **condition** is **fully known**, we have a **necessary** judgment, which may be expressed in the form—'If S is P, it is Q;' 'If a triangle is right-angled, the square on the hypotenuse is equal to the sum of the squares on the other two sides'. When the **condition** is not fully known, or when its operation is counteracted by external influences, we have a **problematic** judgment in the form 'If S is P, it may be Q'; 'if you read carefully, you may pass the examination'; 'if you take medicine, you may recover'. An **assertoric** judgment is a statement of some **fact of experience**, *e.g.* 'This fruit is an orange', or 'all lions are tawny'. Another distinction which follows from the above is that necessary and problematic judgments involve inference, while an assertoric judgment simply records some fact of experience. If this be true, then a necessary judgment cannot be more certain than an assertoric judgment, because inferential knowledge depends upon experience.

From the objective point of view a **necessary** judg-

ment states the operation of some **law,** *e.g.* 'Planets move in elliptical orbits.' It is necessary and universal because it holds good of all planets, known and unknown. It is the task of induction to establish such laws. An **assertoric** judgment is a statement of fact and not the expression of any law, *e.g.* 'All the kings who ruled in France in the 18th century were named Louis', or 'This flower is red'. A **problematic** judgment makes an assertion **without the knowledge of a necessary relation** between the subject and the predicate, *e.g.* 'A seedling rose may be produced different in colour from any roses with which we are at present acquainted.' This judgment implies that there is nothing inherent in roses (or in the laws regulating the production of roses) to render this impossible. So Joseph says that "a problematic judgment is provoked by knowledge; it is problematic because of ignorance." But the above distinctions cannot be regarded as formal. Necessary and problematic judgments, it is apparent, involve reflection, while assertoric judgment is independent of any such reflection. According to Welton, all particular judgments are problematic, all generic universal judgments and hypothetical judgments are apodeictic, all propositions based on mere uncontradicted experience are assertoric. Induction will throw much light upon the distinctions given here.

New Classification of Propositions

The traditional classification of propositions gives us forms which overlap and are not exhaustive. It

New classification of propositions to remedy defects of the traditional scheme.

supposes that every proposition must have a subject, copula, and predicate, which, we have found, is not always required by thought. Thus the traditional classification is not merely inexhaustive but is also confusing. Modern logicians, including Russell, Johnson, Welton and Monahan etc., provide a new scheme of c'assification to remedy the shortcomings of the traditional one. This new scheme provides us with three main classes, *viz*. **simple, complex** and **general.**

1. (**Simple** propositions are: (a) Subject—attribute, *e.g.* 'This mango is sweet,' 'this house is beautiful,' etc. These are **perceptual** propositions, attributing some character to some definite object. (b) **Relational** propositions, which may be either (i) two-termed, etc., *e.g.* 'John met James,' 'Othello killed Desdemona,' 'my house is between a garden and a river,' etc., or (ii) class propositions, *e.g.* 'Roses are included in the class of flowers,' or 'the Ganges is a river,' etc. These propositions may have negative forms.

2. **Complex** propositions are combinations of propositions. They have four forms: (a) **Conjunctive** propositions, *e.g.* 'James and John are coming', 'the man got up and ran to see the result of the game,' etc. A conjunctive proposition contains two or more propositions combined by 'and.' (b) **Implicative** propositions, *e.g.* 'If you come, I shall be happy'; 'if you run, you may fall down', etc. Here the propositions are combined into a complex one through the relation of implication. (c) **Disjunctive**

propositions contain two or more propositions which cannot all be true at the same time. The form of such propositions is 'S is not both P and Q'; 'Gandhi cannot be both an Indian and a German,' 'a definite portion of a river cannot be both wide and narrow', etc. (d) **Alternative** propositions are a combination of two or more propositions, one of which must be true, *e.g.* 'This man is either an Indian or a non-Indian,' or 'this flower is either sweet-scented or is devoid of sweet scent,' etc. Some complex propositions may also have a negative form.

3. **General** propositions may be either (a) about the **whole of a class,** *e.g.* 'All men are mortal,' 'all birds have wings,' etc ; or (b) about **some of a class,**—'Some scholars are unsocial,' 'some books are interesting', etc. General propositions may also have negative forms. We have already noted that 'some' does not exclude 'all', though it does exclude 'none'. 'Some' means 'at least one' but it is compatible with 'all'. Some logicians wrongly suppose that 'some' excludes both 'all' and 'none.'

The following chart, provided by Welton and Monahan, illustrates the above distinctions :—

Propositions

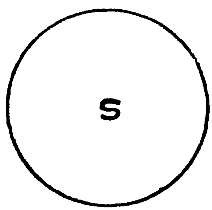
(1) Simple	(2) Complex	(3) General
(a) Subject—attribute	(a) Conjunctive	(a) All
(b) Relational	(b) Implicative	(b) Some
(i) Two-termed etc.	(c) Disjunctive	
(ii) Class propositions	(d) Alternative	

CHAPTER III

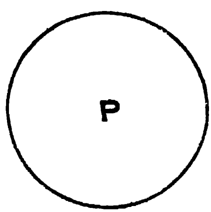
DIAGRAMMATIC REPRESENTATION OF PROPOSITIONS

It is often useful to represent the **relation** between the **subject** and **predicate** of propositions by means of geometrical **diagrams.** We give below the scheme of diagrams devised for this purpose by the Swiss logician, Euler, which are usually called by his name **(Euler's diagrams).**

The fundamental principle of these diagrams is that **all** the **individuals included** in any **class,** or **denoted** by a **name,** are represented by a circle. Thus 'all the individuals included in the class S' (or 'all the individuals denoted by S') are represented by

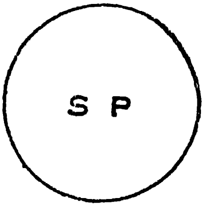


and 'all the individuals included in the class P' (or 'all the individuals denoted by P') are represented by

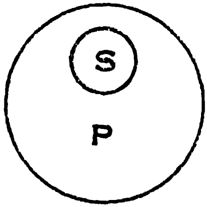


Then all possible relations between the class S and the class P are represented by the following **five** diagrams:—

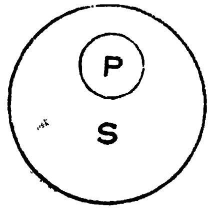
(i)



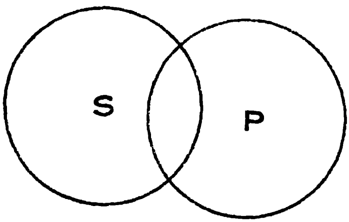
(ii)



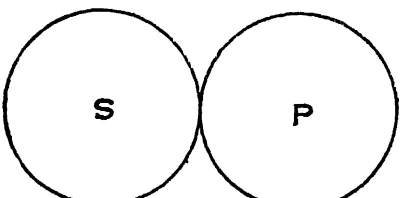
(iii)



(iv)



(v)



In diagram (i), the two circles **coincide** ; *i.e.* the class S and the class P are **co-extensive** ;

diagram (ii) indicates that **all** members of the class S **are included** within the larger class P ;

diagram (iii) indicates that all members of the class P are **included** within the larger class S ;

in diagram (iv) the two circles **overlap**, indicating that **some** (but not all) members of class S are **identical** with **some** (but not all) members of class P ;

in diagram (v) the two circles are completely **outside each other**, indicating that the two classes S and P have **no members in common.**

Let us see how these **diagrams apply** to the **fourfold scheme** of propositions.

(i) Where the classes S and P are co-extensive, it is obviously true both that 'all S is (all) P' and that 'some S is (some) P' : *i.e.* diagram (i) **represents certain A propositions and certain I propositions.** *E.g.*, (A) all equilateral triangles are equiangular triangles; (I) some members of the Cabinet have divulged a Cabinet secret.

(ii) Where all the members of class S are included within class P, it is obviously true both that 'all S is (some) P' and that 'some S is (some) P' : *i.e.* diagram (ii) also **represents certain A propositions and certain I propositions.** *E.g.*, (A) All men are mortal ; (I) some triangles are rectilineal figures.

(iii) Where all the members of class P are included within class S, it is obviously true both that 'some S is P' and that 'some S is not P' : *i.e.* diagram (iii) **represents certain I propositions and certain O propositions.** *E.g.*, (I) Some ships are steamships ; (O) some Hindus are not orthodox Hindus.

(iv) Where the classes S and P overlap, it is obviously again true both that 'some S is P' and that 'some S is not P' : *i.e.* diagram (iv) also **represents certain I propositions and certain O propositions.** *E.g.*, (I) Some knaves are fools ; (O) Some students are not diligent.

(v) Where the classes S and P have no members in common, it is obviously true both that 'some S is not P' and that 'no S is P' : *i.e.* diagram (v) **represents not only certain O propositions but also all E propositions.** *E.g.*, (O) Some effects are not without a cause ; (E) no human being is infallible.

Conversely,

An A proposition is always represented by
either diagram (i) *or* diagram (ii) ;

An I proposition is always represented by
either (i) *or* (ii) *or* (iii) *or* (iv) ;

An E proposition is always represented by diagram (v) ;

An O proposition is always represented by
either (iii) *or* (iv) *or* (v).

CHAPTER IV

IMPORT OF PROPOSITION

Theories of Predication

In this chapter we are concerned with the import of categorical propositions only, and with this object in view shall examine the various theories of predication which are usually recognised. Without a clear understanding of the nature of predication the treatment of logic becomes impossible. We have thrown out some hints as to the import of proposition in discussing the nature of proposition (Book II, ch. I). Before setting forth the correct view of predication, that is, what the import of proposition really is, we shall give an account of the five different theories recognised by logicians, *viz*. (1) the Predicative view, (2) the Denotative view or the view of Class-inclusion, (3) the Comprehensive view, (4) the Connotative view or the view of Concomitance, and (5) the Indicative view. These are all answers to the problem, what really is **the relation between the subject and predicate of a proposition** ? We may now examine these theories one after another.

The predicative view of proposition is natural and from the psychological point of view most satisfactory.

1. **The Predicative** view.—According to the predicative view the **subject** of a proposition is read in its **denotation**, the **predicate** in its **connotation**, and the relation between them is one of **possession**, that is, every proposition means that the thing or things denoted by the subject **possess** the **attribute** or attributes connoted by the

predicate. Thus the proposition 'man is mortal' implies that human beings possess the attribute of mortality. Similarly 'gold is yellow' implies that the things denoted by the term 'gold' possess the attributes connoted by the term 'yellow'. From the psychological point of view, this theory is most satisfactory, since it represents the natural mode of our thought. Thus when we say that 'all diamonds are combustible' we naturally think of the subject in its denotation, though it connotes attributes as well, and of the predicate in its connotation, though in addition it denotes things. Further the predicative view agrees with the fourfold scheme of propositions. In this form we can make either affirmation or denial about either a definite or an indefinite subject. In favour of this view it may also be pointed out that we quantify denotation and not connotation, and the fact that it is the subject-term only which is quantified according to the traditional scheme also supports this predicative view. The quantification of the subject-term implies that it is read in denotation, whereas the fact that the predicate is not quantified suggests that it should be read in connotation. Further it should be remembered, from the logical point of view, that a substantive is thought of in denotation and an adjective in connotation, though in the former case the denotation may be determined by the connotation. The subjects of propositions are very often substantives, while the predicates are adjectives.

But though the predicate of propositions is not usually read in denotation, yet in some cases the class reference of the predicate is undeniable, *e.g.* in 'all owls are birds', 'all palms are endogens', 'all men are animals' etc.

Not all propositions naturally lend themselves to this interpretation.

Further there may be propositions, though rarely, in which it is natural to take the subject in connotation and the predicate in denotation, as in the proposition 'no plants with opposite leaves are orchids.' Thus it cannot be established that the predicative mode of interpreting propositions is natural in every case.

(2) The **Denotative** or **Class-inclusion** view.—Formal logicians from the time of Aristotle have accepted the denotative or class-inclusion theory of predication.

Denotative view explained.

According to this view both the **subject** and the **predicate** of a proposition are read in **denotation,** and the things denoted by the subject are **included** in those denoted by the predicate. Thus the **relation** between the **subject** and the **predicate** is one of **inclusion.** The proposition 'all owls are birds' means that the class of things denoted by the term 'owls' is included in the class of things denoted by the term 'birds'. Similarly, 'Hindus are Aryans' means that the objects denoted by the term 'Hindus' are included in the objects denoted by the term 'Aryans.' In some cases however the predicate is included in the subject, if they are both read in denotation, *e.g.* 'Some Aryans are Hindus.'[1] Here the denotation of the term 'Aryans' is wider than that of the term 'Hindus', and therefore the former includes the latter. The denotative mode of interpretation requires that the **predicate** should be always taken **collectively.** When we say 'all owls are birds' or 'all men are animals', we do not mean that every owl is any bird

or that every man is any animal? We really mean, in each of these cases, that if we take the predicate collectively then the class denoted by it will include the things denoted by the subject term. It is supposed that this mode of predication is convenient for logical manipulation, and such processes as conversion become possible if both the subject and the predicate are read in denotation. The diagrammatic representation of propositions is based upon the denotative interpretation. Further it is held, as we shall afterwards find, that syllogistic inference is not possible if the subject and the predicate are not both read either in denotation or in connotation. Aristotle's famous 'dictum de omni et nullo' also rests upon the assumption that the subject and the predicate of every proposition are read in denotation.

Remarks upon the class mode of predication.

But this view of predication is neither natural nor ultimate. It ignores the fact that propositions assert a **relation of content**. It does not recognise that every judgment is a single act of thought and is a unity. Though some propositions, in which the subject and predicate are class terms, easily lend themselves to this mode of interpretation, most propositions cannot naturally be so interpreted. Thus such propositions as 'some violets are white', 'some dogs are savage', etc. require that the subject should be read naturally in denotation, but the predicate in connotation. Further some propositions do not express the relation of inclusion, *e.g.*, 'equilateral triangles are equiangular', 'Hyde is Clarendon' etc., because in these cases the sub-

ject and the predicate are co-extensive. Moreover the first of these examples really asserts a relation of content. Further we must remember that in generic judgments connotation determines denotation and not vice versa. Therefore the connotative mode of predication is supposed to be better than the denotative. Hamilton's propositional schedule by quantifying predicates, and Jevons's equational scheme, of both of which we shall give a short account later, rest upon the denotative view of proposition.

(3) **The Comprehensive view.**—Sir W. Hamilton held that "every judgment expresses not only a quantitative relation in extension or denotation between subject and predicate, but also a similar relation in comprehension" (Welton). The **copula** in the former case means 'is contained under' and in the latter case '**comprehends.**' We may explain the matter by an example. The proposition 'all Hindus are Aryans', if viewed in extension, means that the Hindus are contained under the class Aryans. If viewed in intension, the proposition means that the complex notion 'Hindus' **comprehends** or contains the **attributes** common to the class 'Aryans'. We have already, in connection with the denotative view of predication, noted the difficulties which arise, if both the terms are read in extension. Here we may note the difficulties which arise if both the terms are read in intension. If by 'comprehension' we mean all the common attributes of a class, then the proposition given above, and other similar propositions, become analytic, since the predicate

Comprehensive view explained.

Its defects.

simply states some of the attributes which are implied by the subject term, and we have found that analytic propositions do not contribute much to the enhancement of knowledge. If by 'comprehension' we mean connotation, then it is not true that the subject comprehends the predicate, for the connotation of the subject-term, if we mean by it conventional intension, may not be larger than that of the predicate, *e.g.* 'all equilateral triangles are equiangular.' Further this view suggests that intension, like extension, can be quantitatively measured, which is an absurd view. Moreover this, like the denotative, view ignores the unity of judgment and fails to see that judgments usually assert a relation of content. This view has all the defects of the denotative view and others in addition.

(4) The **Connotative** view.—Before stating his own view of predication, Mill criticises a theory common to Hobbes and to certain German thinkers, who hold that a proposition implies a relation between two ideas. This view, according to Mill, is indefensible. He rightly points out that 'fire causes heat' does not mean that the idea of fire causes the idea of heat. We have already pointed out that every judgment is a unity, and is a single idea predicated of some aspect of reality or of the whole of it. Similarly Spencer's view that a proposition is a transition from one idea to another cannot be accepted. According to him, 'man is mortal' means that we pass from the idea of man to the idea of mortal. But the essence of every judgment being

Preliminary criticism of certain other views.

assertion, we cannot rightly say that a judgment is nothing but a psychological transition. Mill rightly criticises Hobbes. According to Hobbes every judgment involves a belief that the predicate is the name of that of which the subject is the name. Thus according to him, 'man is mortal' means that 'mortal' is the name of that of which 'man' is the name. Such an interpretation requires that the extension of the predicate should be equal to that of the subject. But this is not always true. 'Mortal' has a wider extension than 'man.' Such a mode of interpretation can be true only of verbal propositions, *e. g.* 'Cicero is Tully.' Besides, it is connotation that determines denotation and not vice versa.

An explanation of the connotative view.

So Mill argues that both the **subject** and the **predicate** of a proposition, except in the case of some singular propositions, should be read in **connotation**, and the relation between them is one of **agreement** or **concomitance**. When the proposition is negative, the relation is one of disagreement. Thus the proposition 'man is mortal' means that whatever has the attributes connoted by 'man,' has also the attributes connoted by 'mortal,' that is, the attributes connoted by 'man' are in agreement with the attributes connoted by 'mortal'. This view is better than the comprehensive view, since it does not hold that the attributes connoted by the subject term comprehend or include the attributes connoted by the predicate term. It is better than the denotative view, inasmuch as it holds that since connotation determines denotation, agreement between the connotation of the subject and that of the predicate also implies

agreement between the things denoted by the subject and those denoted by the predicate. This view may be better expressed in another way, *viz.* that the **attributes** connoted by the **subject** are an **evidence** or a **mark** of the **attributes** connoted by the **predicate.** 'All S is P' means that the attributes connoted by S are in agreement with, or a mark of, the attributes connoted by P. 'No S is P' means that the attributes connoted by S are not in agreement with the attributes connoted by P. Similarly 'some S is P' means that sometimes the attributes connoted by S are in agreement with the attributes connoted by P. 'Some S is not P' means that the attributes connoted by S are sometimes not in agreement with the attributes connoted by P. According to Keynes this view is ultimate, because connotation really determines denotation. This view is not affected by the quantitative reading of propositions, since connotation implies denotation.

Mill fails to establish universal and necessary propositions owing to his empirical standpoint.

Mill however wrongly supposes that every proposition asserts a relation between states of consciousness. This is due to his empirical standpoint. He reduces all things and attributes to sensations or states of consciousness. Thus his universal propositions are never universal in the strict sense, being neither certain nor necessary. The necessity and universality of a judgment depend upon the relation of content apprehended by the mind. So Mill reduces all propositions to five kinds of relation, *viz.* (1) Existence, *e. g.* 'matter exists'; (2) Co-existence, *e. g.* 'matter is extended' or 'this table is beside that chair'; (3) Sequence, *e.g.* 'the death of Socrates followed his

condemnation'; (4) Causation, *e.g.* 'the drinking of water quenches thirst'; (5) Resemblance, *e.g.* 'the colour of milk is like that of snow.' Mill thinks that such relations between the terms of a proposition can be established by invariable and uncontradicted experience, that is, upon enumeration of instances. But since experience cannot establish the necessary and universal propositions demanded by knowledge, we cannot rest satisfied with the mere empirical standpoint. We can have necessity and universality when our experiences harmonise with the contents of reality. However, Mill's view is a move in the right direction, since it recognises some sort of relation of content.

The Indicative view of predication is unnatural and almost useless.

(5) The **Indicative** view.—According to this view the **subject** is read in **connotation** and the **predicate** in **denotation.** This is the reverse of the predicative view. 'All S is P', according to this view, means that the attributes connoted by S indicate the presence of something belonging to the class P. Very few propositions can be naturally read in this way. But some instances may be given: 'No plants with opposite leaves are orchids,' 'all that glitters is not gold' are examples in point. If we examine the second proposition we find that the subject here is attributive, while the predicate is a substantive. But this view of predication is most unnatural and does not serve any useful purpose. We cannot even logically manipulate propositions on the basis of this theory of predication. We may, therefore, summarily dismiss it.

The Reasonable view of Predication

We may point out that if existence is taken in a wide sense to include the world of mythology, of fiction, of mind, of physical reality, it may be asserted with certainty that **every proposition asserts existence.** The terms of a proposition are names which stand either for things or for attributes, and they must exist either in thought or in some other sphere of the universe. So James says, "In the strict and ultimate sense of the word 'existence,' everything which can be thought of at all exists as some sort of object, whether mythical object, individual thinker's object, or object in outer space and for intelligence at large." A universal affirmative proposition, *e.g.* 'All S is P,' asserts that S and P belong to **the same sphere of existence** or universe of discourse. A universal negative proposition, *e. g.* 'No S is P,' asserts that S and P do not belong to the same sphere of existence. A particular proposition, whether affirmative or negative (*e.g.* 'some S is P' or 'some S is not P') asserts that S and P may or may not belong to the same sphere of existence. Besides, if we look at the proposition as a whole and not at its constituent terms, we also find that since every proposition is either true or false, it must assert either some aspect of reality or the whole of it. Every judgment asserts a relation of content, and must be consistent with the contents of reality in order to be true. Every proposition therefore is adjectival in nature, and is a single act of thought which is predicated of existence or the real world, which is substantial

The ultimate import of propositions explained.

in character. In perceptual propositions the reference to reality is direct, *e.g.* 'This flower is red.' In the case of such categorical judgments as are abstract and universal, *e. g.* 'All triangles have their three angles equal to two right angles,' and hypothetical propositions, *e.g.* 'If a triangle is right-angled then the square on its hypotenuse is equal to the sum of the squares on the other two sides,' the reference to reality is indirect. We may however point out that for convenience of logical manipulation we may break up the unity of proposition into its constituent terms, and read them either in extension or in intension.

The Existential Scheme of Classification. *

Our treatment of propositions has shown that every proposition has reference to existence. Some logicians accordingly have given us an **existential scheme** of propositions. We may briefly point out its leading features. We have found that the four main theories of predication are (1) the predicative, (2) the denotative, (3) the connotative, and (4) the indicative theory. The comprehensive theory is in a way an amalgam of the denotative and connotative views. Every affirmative proposition and every universal negative proposition, if reduced to the existential mode of interpretation, yield four different types of proposition in accordance with the four different theories of predication. Thus the universal negative proposition 'No S is P' may be reduced to the

* If the following paragraphs appear difficult to students, they need not attempt to master the details, but may leave them after reading once.

following four existential propositions : (1) 'There is no individual belonging to the class S and possessing the attributes connoted by P' (from the standpoint of the predicative view) ; (2) 'There is no individual common to the two classes S and P' (from the standpoint of the class-inclusion theory) ; (3) 'The attributes connoted by S and P respectively are never found conjoined' (from the standpoint of the connotative view) ; (4) 'There is no individual possessing the attributes connoted by S and belonging to the class P' (from the standpoint of the indicative theory). Similarly we may illustrate the existential scheme of propositions by taking an I proposition. The proposition 'Some S is P' may be reduced to the following four forms : (1) 'There are individuals belonging to the class S and possessing the attributes connoted by P' (predicative view) ; (2) 'There are individuals common to the two classes S and P' (denotative view) ; (3) 'The attributes connoted by S and P respectively are sometimes found conjoined' (connotative view) ; (4) 'There are individuals possessing the attributes connoted by S and belonging to the class P' (indicative view). The existential scheme may be illustrated by other forms of propositions. But for our purposes the above is sufficient.

We may however point out that there is no need to adopt such a scheme of propositions. If we remember that every proposition has reference to existence, that suffices for the understanding of the import of proposition. Multiplication of propositional schedules makes the study of logic confusing.

Hamilton's Eightfold Scheme of Propositions Read

According to Hamilton it is the fundamental postulate of logic that whatever is implicit in thought should be explicitly expressed in language. Archbishop Thomson, following Hamilton, holds that in thought we quantify not merely the subject of a proposition but also the predicate. So like Hamilton he quantifies the predicate and gives us eight forms of proposition in place of the four traditional forms. They are (1) All S is all P (U); (2) All S is some P (A); (3) Some S is all P (Y); (4) Some S is some P (I); (5) No S is some P (η); (6) No S is any P (E); (7) Some S is not some P (ω); (8) Some S is not any P (O). He expresses these eight forms of proposition by the following symbols :—SUP, SAP, SYP, SIP, SηP, SEP, SωP, SOP. We need hardly mention that Hamilton's classification of propositions rests upon the denotative theory of predication, because he reads both the subject and the predicate in denotation.

Hamilton's eightfold scheme of proposition illustrated.

Hamiltonians suppose that the eightfold scheme is simple. It reduces all propositions to equations. Instead of writing 'All S is all P' we may write 'All S = all P' and so on. Further it has the advantage of making every proposition simply convertible. It is also supposed to simplify the rules of syllogism and does away with the need of the four figures. Similarly the scheme of Hamilton dispenses with the special rules of syllogism and is supposed to yield 36 valid moods instead of 19. Hamilton wanted to revolutionise logic.

Advantages claimed for the Hamiltonian Scheme.

Criticism of the eightfold scheme of propositions.

We may however point out that the main basis upon which the eightfold scheme rests is indefensible. Hamilton makes the psychological assumption that we quantify predicates in thought. In discussing the predicative theory we have pointed out that we naturally read the subject in denotation and the predicate in connotation, and from the psychological point of view the predicative view of predication is most satisfactory. Thus the assumption upon which the Hamiltonian scheme rests falls to the ground. It does not really simplify syllogistic rules, and introduces confusion in thought. Further, U and A propositions can each be well expressed by two propositions consistent with the traditional fourfold scheme. Thus 'All S is all P' is equivalent to the two propositions 'All S is P' and 'All P is S.' Similarly 'All S is some P' is equivalent to the two propositions, 'All S is P' and 'Some P is not S.' Further, 'some' being indefinite, the Hamiltonian schedule does not really give us equations. 'Some S is some P' does not tell us which S is which P. The only advantage of this scheme is that it makes simple conversion possible. We may also point out that 'some' may mean 'some only', excluding 'all' and 'none,' or it may mean 'some at least,' excluding 'none' but not 'all.' Whatever meaning we give to it, we cannot have more than five forms. So the Hamiltonian eight forms are not merely redundant but absurd as well. If 'some' means 'some only,' then the assertion of A is not inconsistent with the assertion of η. Similarly the assertion of Y implies the assertion of O. Similarly if 'some'

means 'some only and not all,' then U becomes consistent with ω—'some but not all S is not some but not all P' is not inconsistent with 'All S is all P'—which is absurd. Thus we get a fivefold scheme, *viz*. U (or ω), A (or η), Y (or O), I, E. Similarly if 'some' means 'some at least,' then also we have a fivefold scheme of propositions. In that case Y and A cease to be independent forms. Similarly η and O no longer remain independent forms. ω ceases to be significant at all because it denies nothing. It becomes consistent with all affirmative forms. Thus the forms are U, A, O, I, E. U, A and I may be true together, and so may O and E. Thus we find that the eightfold scheme cannot be established. If the above is true, then propositions cease to be equational. Further, every denotative view of predication ignores the fact that propositions imply a relation of content. So the fourfold scheme is better than the eightfold scheme. (For details readers may consult Keynes's book on Formal Logic).

The eightfold scheme of quantitative relation can be reduced to five forms only.

Brief Account of Jevons's Equational Schedule of Propositions

Jevons attempts to reduce all propositions to logical or rather mathematical equations. The equational scheme of propositions, as we have already found, rests upon the denotative theory of predication. Equations may be of three kinds: (1) Simple, (2) Partial, (3) Limited. The following is an example of simple equation: 'All exogens are dicotyledons' is equivalent

The three kinds of logical equation.

to 'Exogens = Dicotyledons'. The symbolic form is S = P, which may be reduced to the two propositions 'All S is P,' 'All P is S'.

Partial identity.—'All men are mortal' is equivalent to 'Men = Mortal men'. The symbolic form is S = SP. Thus from 'All S is P' we may have two equational propositions, S = SP, SP = S.

Limited identity.— This form is not distinct from simple identity. The following is an example: 'All equilateral triangles are equiangular', which may be equationally expressed as 'Equilateral triangles = Equiangular triangles'. The symbolic form is VS = VP, which means that wtihin the class V, S = P.

We may now see how A, E, I and O propositions can be expressed mathematically:

Reduction of propositions to equational forms.

'All S is P' may be expressed by S = SP or by SP′ = 0 (P′ standing for not-P). 'Some S is not P' may be expressed by S > SP or by SP′ > 0.

'No S is P' may be expressed by SP = 0 or by S = SP′. Similarly 'Some S is P' may be expressed by SP > 0 or by S > SP′.

The view that proposition is equivalent to logical equation is psychologically indefensible. It cannot be grasped easily. We should not break away from the traditional schedule of propositions more than is necessary, though the equational scheme of propositions is not unworkable. We have already remarked

Some remarks on the equational scheme of propositions.

that the multiplication of propositional schedules leads to unnecessary difficulties. Logic is not mathematics and propositions need not be expressed by equations. (For details readers may consult Jevons.)

BOOK III

INFERENCE

CHAPTER I

IMMEDIATE INFERENCE—OPPOSITION AND EDUCTION

General Remarks on Inference

Inference is a mental act by means of which the truth of a proposition is derived from the truth of other propositions. The passage from judgment to inference is not abrupt. In judgment we directly refer some idea which is clear to us to some content or element of reality. In inference also there is the same reference to some content of reality, but here the reference to reality is not direct but is made through the medium of other judgments. Joseph defines inference as "a process of thought which, starting with one or more judgments, ends in another judgment whose truth is seen to be involved in that of the former." Every inference involves a movement of thought, since in every inference the truth of a proposition is derived from other propositions. The name 'inference' is given both to the process of thought and to the conclusion thus derived.

The process of thought involved in inference yields as its result the derived proposition, which is called the conclusion. The propositions from which the conclusion is derived are called the **premises** or the **data**. Thus

in the inference 'No men are perfect, Socrates is a man, therefore Socrates is not perfect,' the first two propositions are the premises or data, while the third is the conclusion. Thus every inference requires some premise or premises which imply the conclusion. The implications of the data of inference are unfolded in the conclusion. Thus the conclusion necessarily follows from the premises. But though this is true, the conclusion must not be a repetition of the implications of the premises ; it must impart some new information without which inference fails to contribute to the advancement of knowledge. Inference therefore requires that the **conclusion** should **follow** necessarily from the **premises** and at the same time **go beyond** them ; that is, it must be in the premises and yet outside them. This is the paradox of inference. So Bosanquet defines inference as consisting in "asserting as fact or truth, on the ground of certain given facts or truths, something which is not included in those data." Though the premises are the ground of inference, they yield a new result as the consequence or conclusion.

Since inference is a mental act, it involves **construction,** inasmuch as the elements in the premises have to be synthesised or brought together to yield the conclusion. So according to Bradley inference involves synthesis and perception. Since inference brings out the nature of proof, just as proposition does that of assertion, there is **demonstration** in inference. The conclusion of an inference is **proved** by the premises, which are the evidence. Proof or demonstration therefore consists in seeing or perceiving that the synthesis or connection

established in the conclusion is involved in or is implied by the premises. On this ground Bradley's remark that inference involves synthesis and perception holds good. Johnson compares inference with assertion. Assertion of a proposition is a mental act, so is inference. Inference is related to implication, according to Johnson, in the same way as assertion is related to proposition. He means that an inference unfolds the implication of premises just as an assertion unfolds the nature of a proposition. Inference however, according to him, being a mental act, is distinct from the relation of implication, just as an assertion, which is a mental act, is distinct from a proposition, which is a statement about some fact. Since the validity of the conclusion of an inference follows from the given premises, every inference admits of formal treatment. In every inference the validity of the conclusion depends upon its being consistent with the premises. Formal logic requires only formal consistency, that is, it only demands that in inference the **conclusion** must be **consistent** with the **premises** and must follow from them. It does not enquire whether the premises are true or valid ultimately. But in the wider sense of validity, the validity of the conclusion depends as much upon the validity of the premises as upon its being consistent with them.

Different kinds of inference, *viz.* Immediate inference, Syllogism and Inductive inference.

Inference may be either **immediate** or **mediate. Mediate** inference may be either **deductive,** that is, **formal,** or **inductive,** that is, **material.** In **immediate** inference the conclusion follows from **one premise** only, *e.g.*, 'All monkeys are animals (premise), therefore some animals are

monkeys' (conclusion); 'all men are rational (premise), ∴ no men are irrational' (conclusion), etc. In such an inference the implication of one premise is unfolded in the conclusion. **Syllogism** or ratiocination is the name of the formal or **deductive mediate** inference. In syllogism a conclusion is drawn from **two** given premises, *e.g.*, 'All stones are hard, marble is a stone, therefore marble is hard'. In formal or deductive inference we do not enquire whether the premise or premises are true or false. To establish the validity of a certain piece of deductive inference it is sufficient to show that the conclusion legitimately follows from the given premise or premises. Kant calls immediate inference the "syllogism of the understanding", and he contrasts it with mediate syllogistic inference, that is, syllogism proper, which he calls the "syllogism of reason". But modern logicians give the name syllogism only to deductive mediate inference. **Induction** is another form of **mediate** inference, in which a **general conclusion** is drawn from **observed instances**, *e.g.*, 'this cow is herbivorous, that cow is herbivorous, and all the cows observed are herbivorous without any exception, therefore all cows are herbivorous'.

In **deductive** inference the **conclusion** can **never** be **more general** than the **premises**, but in **inductive** inference the conclusion is **always more general than** the premises. In deduction we start with general premises, and establish a conclusion which must be either equally general or less general, but never more general, than the premises. In induction we start with observed particular instances, and from them pass to a conclusion which is

always more general than the data.) The direction in which thought moves in deductive inference is the opposite of that in which it moves in inductive inference. Though this distinction is true, we shall find in the course of our discussion that deduction and induction are complementary, and neither can do without the other. We do not subscribe to the view that all inference, whether deductive or inductive, can be reduced to syllogism or ratiocination, which according to some logicians is the only process of reasoning. We shall find as we proceed that though induction and deduction are allied to each other, they represent two distinct processes of thought. In this volume we shall discuss only the implications of deductive inference, and shall take up inductive inference in the second volume.

Immediate inference properly regarded as inference.

(Some logicians hold that immediate inference is not inference proper. According to Mill, immediate inference is nothing but equipollency or equivalence of propositions. He means that immediate inference involves nothing but verbal alteration. Though it is true that in some cases of immediate inference the movement of thought is slight, yet since immediate inference unfolds the implication of a certain given premise, it can properly be regarded as inference. "The step from premise to conclusion in an immediate inference is small, but this does not prove that it is no step at all" (Welton). In this chapter we shall deal with immediate inference under the two main heads of **Opposition and Eduction.**)

Opposition of Propositions

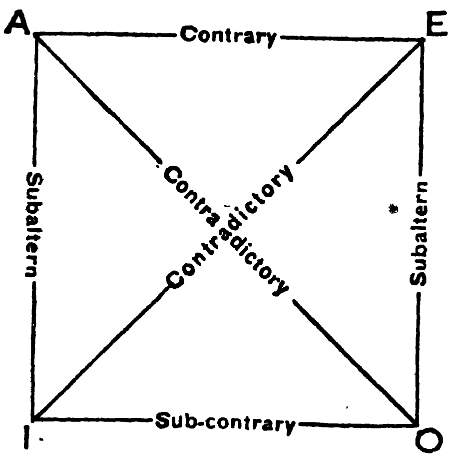
"**Two propositions** are technically said to be **opposed** to each other when they have the **same subject** and **predicate** respectively, but **differ** in **quantity** or **quality** or both" (Keynes). "Opposed propositions differ in form but refer to exactly the same matter, that is, to the same things, at the same time, and under the same circumstances" (Welton).

Meaning of opposition.

Four kinds of opposition of propositions are ordinarily recognised in logic, *viz*. Contrary opposition, Sub-contrary opposition, Contradictory opposition and Subalternation. If two **universal** propositions having the same subject and predicate differ only in **quality**, we have **contrary** opposition. Thus A—All S is P, and E—No S is P, are opposed to each other, and such an opposition is called contrary opposition. If two **particular** propositions having the same subject and predicate differ only in **quality**, we have **Sub-contrary** opposition. Thus the opposition between I—Some S is P, and O—Some S is not P, is sub-contrary opposition. We have **contradictory** opposition, when two propositions having the some subject and predicate differ both in **quality and quantity.** Thus the opposition between A—All S is P, and O—Some S is not P, as well as between E—No S is P, and I—Some S is P, is contradictory opposition. We have the opposition of **Subalternation** between two propositions when they differ in **quantity** only but **not in quality.** Thus the opposition between A—All S is P, and I—Some

Four kinds of opposition.

S is P, and between E—No S is P, and O—Some S is not P, is subaltern opposition. In **subaltern** opposition the **universal** proposition is called the **subalternant** or subalternans, and the corresponding **particular** proposition is called **subalternate** or subaltern. Thus the proposition A is the subalternant of the proposition I, while I is the subalternate of A. Similarly the proposition E is the subalternant of the proposition O, which is its subalternate or subaltern. These relationships are exhibited in the following diagram, known as the Square of Opposition:



"The doctrine of opposition may be regarded from two different points of view, namely, first as a relation between two given propositions; and, secondly, as a process of inference by which one proposition being given either as true or as false, the truth or falsity of certain other propositions may be determined" (Keynes). We have shown how the propositions A, E, I and O are

Illustration of opposition as involving inference.

opposed to each other. We may now point out how opposition involves inference.

When two propositions are opposed to each other as **contrary** propositions, if **one** of them is **true**, the **other must be false,** but **both** the propositions **may be false** at the same time. If the proposition 'Everything in Aristotle is true' is true, then the proposition 'Nothing in Aristotle is true' is false. Similarly if the latter proposition, which is an E proposition, is true, then the former proposition, which is an A proposition, must be false. But it does not follow that if one of the above propositions is false the other must be true, for both the propositions 'Everything in Aristotle is true' and 'Nothing in Aristotle is true' may be false at the same time. Between two **sub-contrary** propositions the relation is such that if **one** of them is **false,** the **other must be true,** but **both may be true** at the same time. Thus if the proposition 'Some men are God-fearing' is false, then the proposition 'Some men are not God-fearing' must be true, and vice versa. But both the propositions 'Some men are God-fearing' and 'Some men are not God-fearing' may be true at the same time. It will be apparent now that contrary opposition rests upon the principle of contradiction, which may be symbolically expressed as 'A is not both X and Y.' From this it follows that if A is X it is not Y, and if it is Y it is not X. But sub-contrary opposition rests upon the principle of excluded middle, which may be symbolically expressed as 'A is either X or Y', which means that if A is not X it is Y and if it is not Y it is X.

The relation between **contradictory** propositions is

such that if **one** of them **is true, the other must be false,** and if **one** of them is **false,** the **other must be true.** Thus contradictory opposition rests upon the principles of contradiction and of excluded middle at the same time. Thus if the A proposition 'All Mussalmans are brave' be true, then the O proposition 'Some Mussalmans are not brave' must be false. Again if the former proposition be false, then the latter must be true. Similarly if the E proposition 'No philosophers are practical men' be true, then the I proposition 'Some philosophers are practical men' must be false. In the same way if an E proposition is false, then the corresponding I proposition must be true.

Between two **subaltern** propositions the relation is such that if the **universal** proposition is **true,** then the corresponding **particular** is **also true,** but **not conversely**; that is, if the **particular** proposition be true, then the corresponding **universal** proposition **need not be true.** Again, of two subaltern propositions, if the **particular** be **false,** then the corresponding **universal must be false,** but not vice versa; that is, if the **universal be false,** then the corresponding **particular need not be false.** Thus if the A proposition 'All lovers of virtue are lovers of angling' be true, then the corresponding I proposition, 'Some lovers of virtue are lovers of angling,' must also be true. But if the I proposition 'Some men are happy' be true, then the corresponding A proposition 'All men are happy' need not be true. Again if the proposition 'Some cows are carnivorous' be false, then the proposition 'All cows are carnivorous' must also be

false. But if the proposition 'All men are happy' be false, then the proposition 'Some men are happy' need not be false. Similarly we may illustrate the relation between E and O propositions. Let us take the two propositions 'No graduates are illiterate' (E) and 'Some graduates are not illiterate' (O). It will be apparent that if the above E proposition be true, then the O proposition is also true. But if the proposition 'Some flowers are not red' (O) be true, the proposition 'No flowers are red' need not be true. Again if the proposition 'Some men are not two-footed' be false, then the proposition 'No men are two-footed' must also be false. But if the proposition 'No men are musical' be false, then the proposition 'Some men are not musical' need not be false.

True opposition exists between propositions which cannot be true together, that is, which are incompatible. If we take opposition in this sense, then strictly speaking there are only two kinds of opposition, *viz.* contrary opposition and contradictory opposition, that is, opposition between A and E and also between A and O and between E and I. In that case sub-contrary opposition and subaltern opposition cannot strictly speaking be regarded as opposition, because I and O may be true together, and in the same way A and I as well as E and O can be true at the same time. So Bosanquet regards sub-contrary opposition as no opposition at all. But in our discussion we have followed the usual practice of logicians. Though sub-contrary opposition and subalternation are not oppositions proper, yet a distinction exists between sub-contraries

and subalterns which ought to be exhibited.) Opposition has come to include all relations between propositions having the same subject and predicate but differing in form, whether such relations are incompatible or compatible. Though contrary and contradictory oppositions are the main types of opposition, yet since contradiction involves contrariety and also goes beyond it, it may be regarded as more effective than contrariety to disprove the truth of a proposition. Contrary propositions may admit of a mean between them, as the propositions 'All men are happy' and 'No men are happy' do, and therefore both may be false. But contradictory propositions allow no such mean and therefore one of them must be true and the other false, *e.g.* 'All negroes are black' and 'some negroes are not black' So **contrary** propositions **cannot both** be **true** at the same time; but **contradictory** propositions can **neither both** be **true nor both** be **false** at the same time.

We may now provide a chart to show how the doctrine of opposition is a doctrine of immediate inference:

A being given true, E is false, I true, O false.

E being given true, A is false, I false, O true.

I being given true, A is unknown or doubtful, E false, O unknown or doubtful.

O being given true, A is false, E unknown or doubtful, I unknown or doubtful.

A being given false, E is unknown or doubtful, I unknown or doubtful, O true.

E being given false, A is unknown or doubtful, I true, O unknown or doubtful.

I being given false, A is false, E true, O true.

O being given false, A is true, E false, I true.

Opposition of singular propositions.

We may now note in passing that between two **singular** propositions having the same subject and predicate but **differing** in **quality,** the only kind of opposition that can exist is **contradictory** opposition; *e.g.*, 'Socrates is honest' (A) and 'Socrates is not honest' (E). Such an opposition is called **secondary opposition.** In this case if we affirm one of the propositions we are bound to deny the other, and if we deny one we are bound to affirm the other, that is, if one is true the other is false and *vice versa*. Keynes points out that even in the case of singular propositions the ordinary squares of opposition can be obtained by quantifying the predicates. Thus of propositions in which 'Browning' is the subject and 'obscure' the predicate, we have the following four forms:—'Browning is always obscure' (A), 'Browning is never obscure' (E) (contrary), 'Browning is sometimes obscure' (I) (subalternate), 'Browning is sometimes not obscure' (O) (contradictory). But if a singular proposition is interpreted in different ways by quantifying the predicate, it ceases to be a singular proposition, because the subject no longer remains a *whole indivisible*. Though this is true, such manipulation of singular propositions is not useless, because 'Browning is obscure' does mean that he is obscure at times but not always.

Opposition of propositions is a formal distinction and not a material one, as has been clearly brought out by

the previous discussion. We may also remark that since the truth of I follows from the truth of A and the truth of O from that of E, if the two contrary propositions A and E were true together, it would follow that the contradictories A and O as well as E and I became true together, which is absurd. We have previously held that contradictory opposition exists between A and O and also between E and I. But if 'Socrates is honest' and 'Socrates is not honest' are contradictories, then A and E may be contradictories at times. So we may say that two propositions having the same subject and predicate, whether they differ both in quality and quantity or only in quality, may be contradictories, if the positing of one involves the sublating of the other and the sublating of one involves the positing of the other. A complex proposition can be contradicted by another complex proposition. Thus the proposition 'He came here and saw me' can be contradicted by the complex proposition 'Either he did not come here or he did not see me'. Similarly if we take the meaning into consideration and not the form only, then two affirmative propositions can be contraries, *e.g.* 'This flower is green' (A) and 'this flower is red' (A). If one of the propositions is true then the other is false, but both may be false, since the flower may be yellow. We have already remarked that a proposition is not denied by its contrary as effectively as by its contradictory.

Contradiction and contrariety further considered.

The doctrine of opposition has been extended by traditional logic to **hypothetical** and **disjunctive** proposi-

Hypothetical propositions reduced to the four traditional forms, that the doctrine of opposition may be applicable to them.

tions also.) We have already stated that such propositions assert a relation of content and are abstract, and when they are expressed in the denotative form they deviate from the ideal, but still they may be so expressed. Let us first consider the abstract forms and then the concrete forms of **hypothetical** propositions, to find how the doctrine of opposition has been applied to them. The hypothetical proposition may be expressed by the symbol 'If X then Y,' or 'If S is M, it is P.' Then we have the following four propositions :—(A) If X then Y, or If S is M it is P ; (E, contrary) If X then not Y, or if S is M it is not P ; (I, subaltern) If X then perhaps Y, or if S is M it may be P ; (O, contradictory) If X then not necessarily Y, or If S is M it need not be P. We may exemplify the distinction by a concrete example : If the stone is material it occupies space (A) ; if the stone is material it does not occupy space (E) ; if the stone is material it may occupy space (I) ; if the stone is material it need not occupy space (O). Let us now consider the hypothetical proposition when expressed in the denotative form :—

(A) If any S is M that S is always P ;
(E) If any S is M that S is never P (contrary) ;
(I) If an S is M that S is sometimes P (subaltern) ;
(O) If an S is M that S is sometimes not P (contradictory).

A concrete example is : If any man is honest he is always trusted (A) ; if any man is honest he is never

trusted (E); if some men are honest they are sometimes trusted (I); if some men are honest they are sometimes not trusted (O). Of the above, the I and the O propositions can also be expressed as 'If some men are honest they may be trusted' and If some men are honest they need not be trusted.'

Application of doctrine of opposition to disjunctive propositions.

The doctrine of opposition has been extended not only to hypothetical propositions but also to **disjunctive** propositions. Disjunctive, like hypothetical, propositions being abstract, their ideal form is non-denotative, though it is possible to devise equivalent denotative forms. To extend the doctrine of opposition to disjunctive propositions they have been reduced to the four traditional forms. When the alternatives of a disjunctive proposition are not referred to the same subject, it may be symbolically expressed as 'either X or Y,' which is an A proposition, and can have as its negative 'neither X nor Y,' which is an E proposition. But the negative form is not a disjunction at all. In this form we cannot have either an I or an O proposition. When the alternatives are referred to the same subject, we may have the following four forms of disjunction :

(A) S is either P or Q;

(E, contrary) S is neither P nor Q;

(I, subaltern) S may be either P or Q;

(O, contradictory) S need not be either P or Q.

The following concrete example may be given :—

Virtue is either useful or harmful (A);

Virtue is neither useful nor harmful (E) ;

Virtue may be either useful or harmful (I) ;

Virtue need not be either useful or harmful (O).

If a disjunctive proposition is expressed in the denotative form we may have the following propositions :

(A) Every S is either P or Q ;

(E, contrary) No S is either P or Q ;

(I, subaltern) Some S's are either P or Q ;

(O, contradictory) Some S's are neither P nor Q.

The following is a concrete example :—

Every man is either intelligent or foolish (A) ;

No man is either intelligent or foolish (E) ;

Some men are either intelligent or foolish (I) ;

Some men are neither intelligent nor foolish (O).

(If students find the following paragraphs difficult, they may pass on after reading them once).

According to Bosanquet contrary denial has a positive import, but contradictory denial has no such import and is therefore non-significant.

[We have observed that in the strict sense opposition implies that two opposed propositions cannot be true at the same time, and that in this sense contradiction and contrariety are the only types of opposition. Both contradiction and contrariety involve denial, and we may, in passing, consider briefly the nature of denial. According to Bosanquet, contrary denial alone has a positive import, while contradictory denial has no such positive meaning. According to him contrary denial rests upon the

principle of contradiction, which may be expressed by the symbol 'A is not both X and Y', while contradictory negation rests upon the principle of excluded middle, the symbol of which is 'A is either X or Y'. But we have already pointed out that contradictory denial rests as much upon the principle of contradiction as upon that of excluded middle. According to Bosanquet, when A negates E there is a positive ground, but when A negates O, or E negates I, it is a case of pure denial. Thus he says that all significant denial is made by contraries, while contradictory denial is destructive and meaningless. In contrary denial we argue that because all men are mortal, it cannot be that no men are mortal. But when by contradiction 'Some men are not mortal' is denied, no positive assertion is thereby made: when by denying the given proposition we posit the proposition 'All men are mortal', we do so by means of contrary opposition. Thus Bosanquet's argument leads us to the conclusion that there may be pure denial, which is involved in contradictory opposition, and such denial is non-significant; while contrary opposition provides us with denial having a positive background, and such denial alone is significant. He therefore challenges the view of Sigwart, who argues that pure denial is an impossibility and every denial has a positive background.

But contradictory denial is as significant as contrary denial, though less definite.

In criticism of Bosanquet we may point out that contradiction involves contrariety though it goes beyond it. When A negates O or E negates I, there is really a positive element. Thus when the proposition 'Some men are perfect' is denied or negated by contradiction, we surely get the positive assertion that 'No men are perfect.' Similarly when the proposition 'Some men are not mortal' is denied by contradictory opposition, it yields the positive proposition 'All men are mortal'. Similarly when A is denied

by O, or E by I, we thereby assert the truth of O or of I. If this is true, then contradiction as much as contrariety has a positive background and provides significant denial.

There may be a mental attitude of pure denial. But the purpose of negation is always to establish a positive result. Negation leads to determination.

Can there be pure denial ? Johnson argues that there may be a mental attitude of pure denial. Thus a philosopher may deny that matter exists, without making any positive assertion. Such denial is purely subjective. Similarly such a proposition as 'The soul is not blue' or 'Virtue is not square' gives us an example of pure denial. But such a proposition is nonsense and properly speaking is no proposition at all. Johnson further argues that in denying that a particular man is in this room or that a man is doing something now, we may have a mental attitude of pure denial. We agree that there may be a mental attitude of pure denial, but the purpose of denial is to establish some positive assertion in every case, otherwise negation becomes useless. Induction has recourse to negation to establish a positive result. Pure denial for the time being is possible. but such negation clears the way for positive assertion. By establishing that a thing is not this nor that nor that, and so on, we may ultimately determine what it really is. Negation therefore leads to determination and determination involves negation. Further, negation is useful for bringing out the distinction between difference and incompatibility. If anyone asks 'Is Gandhi at Sealdah Station ?' and receives the answer that Gandhi is at Howrah station, he may not understand that the latter station is other than the former station. So to make the matter definite and determinate, it has to be said that Gandhi is not at Sealdah but is at Howrah station. Here by introducing a negation it is clearly shown that Howrah station is other than Sealdah station. We may

conclude with Sigwart that there may be pure denial when it is due to defective knowledge or ignorance, but when denial is due to opposition it is always significant and rests upon a positive ground.]

Eductions

General remarks on eduction.

We have already seen that opposition is a process of immediate inference, as the truth or falsity of one proposition is implied by, or follows from, the truth or falsity of other propositions having the same subject and predicate but differing either in quantity or in quality or in both. We have already remarked that in immediate inference the implication of one proposition is brought out by another proposition, that is, in it the conclusion follows from a single judgment. We may now, having discussed opposition, consider other kinds of immediate inference known as **Eductions.** "Eductions are those forms of immediate inference by which, from a given proposition, accepted as true, we educe other propositions **differing** from it in **subject**, in **predicate,** or in **both**, whose truth is implied by it" (Welton). Eductions are of four main varieties, *viz.* (1) **Conversion,** (2) **Obversion,** (3) **Contraposition,** and (4) **Inversion.** But some material processes of immediate inference are also called eductions. In considering eductions we are not concerned with tautologous propositions such as S is S, or with propositions which are self-contradictory, *e.g.*, S is not S. Every categorical proposition has two terms, the subject and the predicate, which are generally symbolised by S and P. These terms suggest

as their negatives not-S and not-P, which we shall express by the symbols S′ and P′ respectively. So a categorical proposition suggests to our minds four terms, and in every such proposition two of the above terms must be present. Further, as we have already noted, we shall use the symbols S a P for All S is P, S e P for No S is P, S i P for some S is P, and S o P for some S is not P. The proposition P a S will imply All P is S, and so on. The proposition S′ a P′ will imply All not-S is not-P, and so on. With these observations we may now consider conversion.

1. Conversion.

Nature of conversion.

"A proposition is **converted**, when its **subject is made** the **predicate**, and vice versa, its **quality** (affirmative or negative) remaining **unchanged**" (Joseph). Conversion thus requires the terms of the original proposition to be transposed in the conclusion : S, which is the subject of the given proposition, becomes the predicate of the conclusion, and P, which is the predicate of the given proposition, becomes the subject of the conclusion. The original proposition is called the **convertend** and the conclusion is called the **converse**. In converting a proposition we must observe the following two rules : (1) The **converse** must be the **same in quality** as the **convertend** ; (2) No term must be **distributed** in the **converse** if it was not **distributed** in the **convertend**. (But a term distributed in the convertend may be either distributed or undistributed in the converse).

When converted, A becomes I. S a P when convert-

ed yields P i S as the conclusion. From All S is P we get Some P is S by conversion. 'All Mussalmans are brave' (convertend), 'Some brave men are Mussalmans' (converse). In conversion the predicate term, like the subject term, is read in extension. Keynes says, "In the process of converting a proposition, however, the extensive force of the predicate is made prominent, and an import is given to the predicate similar to that of the subject." Thus if the predicate of a proposition is an adjective, it has to be reduced to a substantive before conversion. Thus by converting the proposition 'Socrates is wise' we get the conclusion 'A wise man is Socrates.' The predicate term 'wise' in the original proposition was an adjective, but as the subject of the converse it has become a substantive. Similarly 'All men are mortal,' when converted. becomes 'Some mortal things are men.' 'All monkeys are animals,' when converted, becomes 'Some animals are monkeys.' In this case the predicate term of the original proposition is a substantive and its denotative significance is apparent, and such a proposition can be converted with greater ease than a proposition which has an adjective as its predicate term. We should also remember that to educe one proposition from another we have to reduce the premise to its logical form. Thus the proposition 'A stitch in time saves nine' has to be reduced to its logical form, *viz.* 'All stitches in time are things that save nine stitches,' from which we may draw the conclusion 'some things that save nine stitches are

Conversion of A propositions. Formally we convert A per accidens or by limitation. Sometimes knowledge of fact may enable us to convert A simply.

stitches in time.' By conversion we get an interpretative proposition as the conclusion.

The above examples show that formally we cannot get an A proposition as the conclusion by converting A. If we pass to an A proposition by conversion from an A proposition, we must have transgressed a rule of conversion. Take the proposition All S is P. The first rule requires that by converting it we must arrive at an affirmative proposition as our conclusion. In the given proposition, P, which is the predicate of an affirmative proposition, is not distributed, and the second rule requires that since it is not distributed in the original proposition, it must not be distributed in the conclusion. In conversion the subject of the original proposition becomes the predicate of the conclusion and the predicate of the given proposition becomes the subject of the conclusion. Now P, which is undistributed in the given proposition and is its predicate, must be the subject of the conclusion when converted and must be undistributed there. The conclusion must also be affirmative, the original proposition being affirmative. So the legitimate conclusion becomes Some P is S. Thus we find that by converting an A proposition we get an I proposition as the conclusion. So the conversion of A is said to be per accidens or by limitation. The conclusion and the premise in this case are not equivalent propositions. Conversion of A involves a loss in the fulness of the meaning of the original proposition. By converting 'All monkeys are animals' we get the conclusion 'Some animals are monkeys'. But by reconverting the conclusion we cannot get back to the original pro-

position. 'Some animals are monkeys' when converted becomes 'Some monkeys are animals,' which is not the same as the original proposition 'All monkeys are animals'. Though formally it is not possible to pass from A to A by conversion, yet in some cases, where the subject and the predicate are co-extensive terms, our knowledge of facts may enable us to do so. Thus the proposition 'All equilateral triangles are equiangular' yields the conclusion 'All equiangular triangles are equilateral.' But we cannot even in this case, without knowledge of geometry, have an A proposition as the conclusion purely on formal grounds. Similarly 'Chatham is the elder Pitt' (A) yields the conclusion 'The elder Pitt is Chatham' (A). So also the proposition 'Demosthenes and Cicero are the greatest orators of antiquity' (A) yields the conclusion, 'The greatest orators of antiquity are Demosthenes and Cicero'. "The conversion of propositions may be studied formally, with symbols for terms, but when real terms replace the symbols they must affect the judgment, and our treatment of it in conversion". Formal treatment is sound within its limits, but we have seen in the course of our discussion that form and matter are not separable. What is form in one case is matter in another. We must be careful in converting an A proposition, because very often the conversion of A leads to fallacy. Granted that it is true that all idle men are poor, it is not true that all poor men are idle. Similarly we cannot pass from the proposition 'All unemployed men are unhappy' to the conclusion that 'All unhappy men are unemployed'. So the conversion of A should as a rule be per accidens or by limitation, though in some cases our material knowledge may justify us in

converting A simply, that is, passing to an A proposition from an A proposition by conversion. But if we adhere strictly to the formal rules of conversion we can never convert A simply ; its conversion must in every case be by limitation.

E converts to E. From S e P we get P e S by conversion. No S is P (convertend), No P is S (converse). 'No cows are carnivorous' (premise), 'No carnivorous animals are cows' (conclusion). Simple conversion of E is possible, because an E proposition distributes both S and P and we are justified in distributing them in the conclusion. 'No S is P' means that 'All S's are other than all P's', and therefore we may say justly that 'All P's are other than all S's', that is, from 'No S is P' we can pass to the conclusion 'No P is S'. From the proposition 'No God-fearing men are unhappy' we may pass to the conlusion that 'No unhappy men are God-fearing'. Since the attribute of being unhappy and the attribute of being God-fearing are absolutely incompatible, that is, one is absolutely different from the other, every object denoted by one of the terms is different from every object denoted by the other term. In the case of an E proposition, the subject and the predicate terms being absolutely different, the relation between them is reciprocal. Thus both the convertend and the converse in an E proposition are equivalent propositions, that is, just as we can pass from No S is P to No P is S, so also we can pass from No P is S to No S is P.

E converts to E and therefore the conversion of E is simple. In this case the convertend and converse are equivalent propositions.

I converts to I. Here also conversion is simple. S i P

when converted becomes P i S. Some S is P (convertend), Some P is S (converse). 'Some Hindus are God-fearing' (premise), therefore 'Some God-fearing men an Hindus'. In the case of an I proposition both the subject and the predicate are undistributed, and therefore when we pass from S i P to P i S, there is no transgression of the rules of conversion. Both P i S and S i P are affirmative propositions, and in both of them both P and S are undistributed. 'Some diamonds are black, therefore some black things are diamonds'. As in the conversion of E, so here the premise and the conclusion are equivalent propositions. Though this is true, the conversion of I does not always yield a happy result. From 'some men are teachers' we may pass to the conclusion 'some teachers are men', but here the conclusion suggests that there may be teachers who are not men, which is false. Welton remarks that "When we speak of the simple conversion of I, we do not mean that 'some' denotes the same proportion of the total denotations of the subjects of both convertend and converse."

I converts to I. Here also conversion is simple and the convertend & converse are equivalent propositions.

O cannot be converted. As it is necessary that if the convertend be negative, the conclusion must also be negative, if we convert S o P, we have to pass to a negative conclusion, viz P o S. If we convert 'Some S is not P', we have to establish the conclusion 'Some P is not S'. But this is not possible, for S, which as the subject of a particular proposition is not distributed in the premise, becomes

O cannot be converted. But knowledge of facts may enable us to pass to an O proposition from an O proposition.

distributed in the conclusion as being the predicate of a negative proposition ; but this is forbidden by the second rule of conversion. Thus from the proposition 'some men are not wise' we cannot pass to the conclusion 'some wise things are not men'. Similarly from the proposition 'some animals are not carnivorous' we cannot pass to the conclusion that 'some carnivorous things are not animals'. So also from the proposition 'some men are not teachers' we cannot pass to the conclusion 'some teachers are not men'.

Sometimes however material conversion of O is possible, just as sometimes the simple conversion of A is justified on the ground of material knowledge. Thus from the proposition 'some masons are not freethinkers' we can pass to the conclusion 'some freethinkers are not masons'. Similarly we can pass from the proposition 'some men are not black' to the conclusion 'some black things are not men'. 'Some S is not P' means that the attributes connoted by some S are incompatible with the attributes connoted by all P. It does not mean that all P's are other than all S's. So Some S is not P and All P is S may be true at the same time. The propositions 'Some men are not wise' and 'All wise things are men' may be true together. To sum up the result from the formal point of view, A converts per accidens, E and I simply, and O not at all.

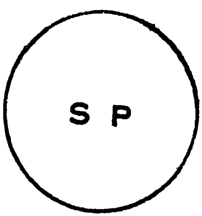
Aristotle tries to show that the legitimacy of conversion can be proved indirectly, but the roundabout way which he adopts to do this appears to us to be absolutely unnecessary. He argues as follows: Suppose the conclusion

We need not, after Aristotle, prove the legitimacy of conversion indirectly.

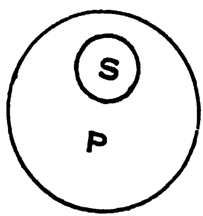
No P is S from the premise No S is P is untrue ; if this is the case, then suppose some individual P, say Q, is S. If this is allowed, then Q becomes both S and P, which is absurd, since in the premise every S excludes every P, and we cannot question the truth of the premise. Therefore the conclusion No P is S from the premise No S is P is legitimate. Having thus shown the legitimacy of the conversion of E, Aristotle deduces from it the legitimacy of the conversion of A and I. We need not further consider the indirect proof of the legitimacy of conversion, since such proof is useless.

The rules for the conversion of A, I, and E propositions may be illustrated by Euler's diagrams :—

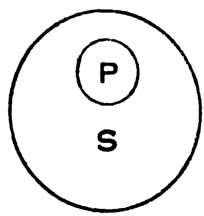
(i)



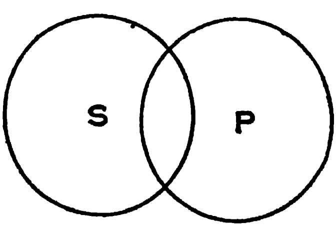
(ii)



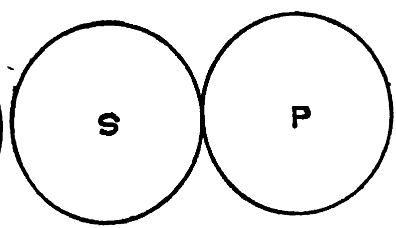
(iii)



(iv)



(v)



A, as we have previously seen, is represented by diagram (i) or (ii). In diagram (i) P and S are obviously co-extensive, *i.e.* we know not only that all S is P but also that all (and therefore also some) P is S. But in diagram (ii) *only some* P is S. But there is nothing in the form of the proposition 'all S is P' to tell us whether it is represented by diagram (i) or diagram (ii); and that being so, all that we are justified in saying about P is what is common to the two cases, *viz.* that *some* P is S. *I.e.*, A converts to I.

An I proposition may be represented by any of the first four diagrams. Here again, in diagrams (i) and (iii) all, and therefore some, P is S, but in diagrams (ii) and (iv) *only some* P is S; and once more, being given that some S is P, we are only justified in affirming of P what is common to all the four possible cases, *viz.* that *some* P is S. *I.e.*, I converts to I.

E is represented always by diagram (v), from which it is plain that the classes S and P have nothing in common, and thus that *no* P is S. *I.e.*, E converts to E.

O is represented by diagrams (iii), (iv) and (v), but here we can say nothing about P that holds good of all three cases, and therefore O cannot be converted. For diagram (iii) shows *all* P as being S, diagram (v) *no* P as being S; and these two are inconsistent with each other.

2. Obversion.

Obversion is a process of immediate inference in which the **inferred proposition (or obverse)**, whilst retaining

The general nature of obversion.

the **original subject**, has for its predicate the **contradictory of the predicate** of the original proposition (or **obvertend)**" (Keynes). Obversion is also called Permutation, Equipollence, Infinitation, Immediate Inference by Privative Conception, or Contraversion. In obversion the original proposition is called the obvertend and the conclusion the obverse. The rule of obversion is, "**Negate** the **predicate** and **change** the **quality**, but leave the **quantity unaltered**" (Welton). By obverting A we get E : S a P (obvertend), S e P′ (obverse) ; All S is P (premise), No S is not-P (conclusion).

A obverts to E.

From the proposition 'Barkis is willing' we get the conclusion 'Barkis is not not-willing' by obversion.' 'All virtuous men are happy' (obvertend), 'No virtuous men are not-happy' (conclusion).

The obverse of E is A : S e P (obvertend), S a P′ (obverse). No S is P when obverted becomes All S is not-P. 'No men are perfect' (obvertend), 'All men are not-perfect' (obverse). When obverted, the proposition 'No triangles are four-sided' yields the conclusion 'All triangles are not-four-sided'.

E obverts to A.

I obverts to O. S i P becomes S o P′ when obverted. Some S is P (obvertend), Some S is not not-P (obverse). 'Some men are rich' (obvertend), 'Some men are not not-rich' (obverse).

I obverts to O.

O obverts to I. S o P becomes S i P′ when obverted.

O obverts to I.

Some S is not P (obvertend), Some S is not-P (obverse). 'Some flowers are not red' (obvertend), 'Some flowers are not-red' (obverse).

A formal contradictory may be replaced by a material contradictory if the meaning remains the same.

We can in obversion replace a formal contradictory by a material contradictory, provided there is no change in the meaning. Thus instead of saying that 'Barkis is not not-willing' we may say that 'Barkis is not unwilling.' Similarly we can replace the proposition 'No men are non-mortal' by the proposition 'No men are immortal'. But the proposition 'No virtuous men are not-happy' cannot be replaced by the proposition 'No virtuous men are unhappy', because 'unhappy' has not the same meaning as 'not-happy' and is not the contradictory of 'happy'. Similarly 'Some men are not-rich' cannot be replaced by the proposition 'Some men are poor,' because 'poor' is not the contradictory of 'rich' and so it does not possess the same significance as 'not-rich'.

Obversion rightly regarded as a mode of inference.

Since every positive proposition can be negatively expressed, Welton, like Mill, holds that obversion should not be regarded as a form of immediate inference, because it does not involve any movement of thought, the obverse being merely the verbal transformation of the obvertend. If this is conceded, then even conversion ceases to be a form of immediate inference, because it involves simply the transposition of terms. We may point out however that when an obvertend is given, it is not always clear what

its obverse should be. Obversion also requires some mental activity, though it is slight.

Relation of obversion to principles of contradiction and excluded middle.

Obversion of affirmative propositions rests upon the principle of contradiction. If a thing is P, it cannot be not-P. Obversion of negative propositions rests upon the principle of excluded middle, according to which if a thing is not P, it is not-P.

Obvertend and obverse are equivalent propositions.

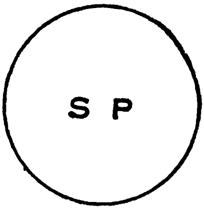
Obversion is a reciprocal process, since we can always pass from a proposition to its obverse and back from the obverse to the original proposition. Thus by obverting All S is P we get No S is not-P as the obverse, and if we obvert this again we can return to the original proposition, the obverse of No S is not-P being All S is P. So in obversion the obvertend and the obverse are equivalent propositions. The obverse always means the same thing as the obvertend, though they differ in expression.

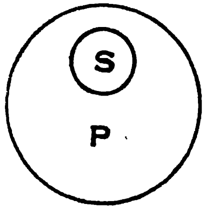
Material obversion is not obversion proper.

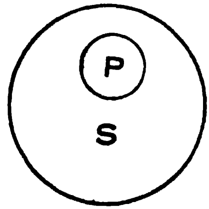
Bain speaks of material obversions, *e.g.* 'Warmth is agreeable' (obvertend), 'cold is disagreeable' (obverse); 'happiness is good' (obvertend), 'misery is bad' (obverse). But such material obversions cannot be regarded as examples of obversion proper, since in them the obvertend and the obverse are both affirmative propositions.

The rules of obversion, like those of conversion, may be illustrated by Euler's diagrams :—

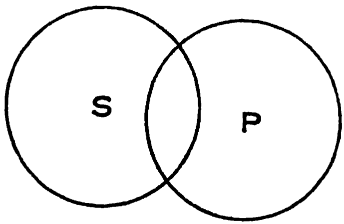
(i) (ii) (iii)



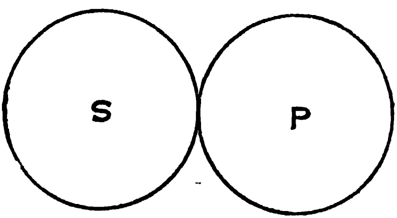




(iv)



(v)



In obversion we want to know the relation between S and not-P.

A propositions are represented by diagrams (i) and (ii), in both of which it is plain that there is nothing in the class S which falls outside the class P ; *i.e.*, No S is not-P.

I propositions are represented by diagrams (i), (ii), (iii) and (iv). In (iii) and (iv) *some* S is identical with some P and is therefore not not-P ; in (i) and (ii), as we have

already seen, No S is not-P, *i.e.*, all (and therefore some) S is other than not-P, *i.e.*, all (and therefore some) S is not not-P. Here we are only justified in stating what is common to all four cases, *viz.* that some S is not not-P.

E propositions are represented by diagram (v). Here evidently, all that belongs to the class S falls outside the class P; *i.e.*, all S is not-P.

O propositions are represented by diagrams (iii), (iv) and (v). We now see that in (iii) and (iv) part of the class S is not included in the class P, *i.e.*, some S is not-P; in (v) we have already seen that all S is not-P, from which it follows that *some* S is not-P; taking what is common to all these cases, we therefore have that some S is not-P.

Obverted conversion.—

If we **first convert** a proposition and **then obvert** it, we get what is known as **obverted conversion**, which is thus a process of reasoning involving both conversion and obversion. Let us take the proposition All S is P. Its converse is Some P is S, and its obverted converse Some P is not not-S. Thus the obverted converse of S a P is P o S′. No S is P (original proposition), No P is S (converse), All P is not-S (obverted converse): thus the obverted converse of S e P is P a S′. Some S is P (original proposition), Some P is S (converse), Some P is not not-S (obverted converse): thus the obverted converse of S i P is P o S′. Some S is not P cannot be converted, being an O proposition; having no converse it can have no obverted converse.

3. Contraposition.

Contraposition is a combination of obversion and conversion. Distinction between contrapositive (or partial contrapositive) and obverted (or full) contrapositive.

"**Contraposition** may be defined as a process of immediate inference in which from a given proposition another proposition is inferred having for its **subject** the **contradictory** of the **original predicate**" (Keynes). Contraposition is not an independent form of immediate inference, but a combination of obversion and conversion. It rests upon the same fundamental principl of thought upon which obversion and conversion rest. The **conclusion** which is obtained by contraposition is called the **contrapositive ;** there is no corresponding name in general use for the original proposition, which however might be (and by some writers is) designated the contraponend. If we first obvert a proposition and then convert it, we get the contrapositive. This process of obversion and conversion is called **conversion by negation** as well as contraposition. Thus the converted obverse of a proposition is called contrapositive, but if we **obvert** this **again** we get what is called the **obverted contrapositive,** or as Keynes calls it, the **full** contrapositive. What we have designated the contrapositive he distinguishes as the **partial** contrapositive. Older logicians gave the name of contrapositive to what we have called obverted contrapositive or full contrapositive, on the ground that since contraposition is a form of conversion, the quality of the premise and of the conclusion should be the same, and that only when the converted obverse is again obverted is the quality of the

conclusion the same as that of the original proposition. But we need not restrict the name contrapositive to what we have called obverted or full contrapositive. We may however make a distinction between simple contrapositive and obverted or full contrapositive. Since the quality of the original proposition is the same as that of the full contrapositive, the latter, as distinguished from the simple contrapositive, provides us with symmetry. We shall regard 'simple' or 'partial contrapositive' as synonymous with 'contrapositive'. Now let us see what results contraposition yields.

Contraposition of A.

S a P—All S is P (original proposition), S e P′—No S is not-P (obverse), P′ e S—No not-P is S (contrapositive, or converted obverse or partial contrapositive), P′ a S′—All not-P is not-S (obverted contrapositive or full contrapositive). Let us illustrate the above by a concrete example : All whales are warm-blooded (original proposition), No whales are not-warm-blooded (obverse), No not-warm-blooded creatures are whales (contrapositive), All not-warm-blooded creatures are other than whales (obverted contrapositive or full contrapositive).

Contraposition of E.

S e P—No S is P (original proposition), S a P′—All S is not-P (obverse), P′ i S—Some not-P is S (contrapositive), P′ o S′—Some not-P is not not-S (obverted contrapositive or full contrapositive). As a concrete example take—No horses are carnivorous (original proposition), All horses are non-carnivorous (obverse), Some non-carnivorous creatures are horses (contrapositive or converted

obverse), Some non-carnivorous creatures are not other than horses (obverted contrapositive or full contrapositive).

Contraposition of O.

S o P—Some S is not P (original proposition), S i P′—Some S is not-P (obverse), P′ i S—Some not-P is S (contrapositive), P′ o S′—Some not-P is not not-S (obverted contrapositive or full contrapositive). This may be illustrated by the following example :—Some flowers are not green (original proposition), Some flowers are non-green (obverse), Some non-green things are flowers (contrapositive), Some non-green things are not other than flowers (obverted contrapositive or full contrapositive).

I cannot be contraposed.

S i P—Some S is P (original proposition), S o P′—Some S is not not-P (obverse). This, being an O · proposition, cannot be converted, and therefore I cannot be contraposed.

Contraposition and obversion are artificial but not on that account useless.

Contraposition, like obversion, is artificial, since it takes recourse to negative terms. Though this is true, contraposition does represent our actual thinking and therefore is not useless. According to De Morgan, if we know that S is P, we also know that not-S is not-P, and not-P not-S. Therefore he argues that contraposition is useless. We do not deny that every form of immediate inference is simple, and if we have the premise we can easily find out what the conclusion is to be. Yet contraposition is not

useless, and we shall discover its utility when we consider inductive inference, in which negative instances are as valuable as positive ones. We should also note that both contraposition and obversion are sometimes able to avoid the use of negative terms. Thus if the premise is 'Anything that is S is P', it may be contraposed as 'Anything that is not P is not S'.

A and O can be contraposed simply, E only by limitation.

Since the contrapositive of S a P is either P′ e S or P′ a S′, and that of S o P is either P′ i S or P′ o S′, A and O are contraposed **simply**, because in them the premise and the conclusion are the same in quantity. But since the contrapositive of S e P is either P′ i S or P′ o S′, the premise and the conclusion in the contraposition of E are different in quantity, and therefore the contraposition of E is per accidens or **by limitation.**

4. Inversion.

General nature of inversion explained. Distinction between partial inverse and full inverse.

Keynes was the first to recognise **Inversion** as a distinct form of eduction. Previously logicians had regarded it as a process of immediate inference not distinct from contraposition. But inversion may rightly be regarded as distinct from contraposition, since the results obtained by the two processes are not the same. We may follow Keynes in defining inversion as "a process of immediate inference in which from a given proposition another proposition is inferred having for its **subject** the **contradictory of the original subject.**" Thus in inversion of a proposition with S as its subject, not-S is to be the subject of the

conclusion. To obtain this result we may begin by converting the original proposition, so that we get S in the predicate; obversion of this new proposition, as we already know, gives us a proposition whose predicate is not-S, and if we can convert this again, we obtain a proposition of which not-S is the subject, which is what we require. We might alternatively start by obverting the original proposition, from which by alternate conversion and obversion we obtain the obverted contrapositive with not-S in the predicate: where conversion of this is possible, we again obtain a proposition whose subject is not-S. We thus find that inversion, like contraposition, is a combination of the two main processes of eduction, obversion and conversion. The original proposition is called the **invertend** and the conclusion the **inverse.** Inversion requires that not-S should be the subject of the conclusion, but its predicate may be either P or not-P. When the subject is not-S and the predicate is not-P we have what Keynes calls **full** inversion, but when the subject is not-S and the predicate is P we have what he calls **partial** inversion. As the **full** inverse is the obverse of the partial inverse, Welton gives it the name of **obverted** inverse. Welton provides the following rule of inversion, which accords with the description given above—"**Convert** either the **obverted converse** or the **obverted contrapositive.**" Whether we begin with obversion or with conversion depends upon the circumstances of the case, as will be apparent from the examples below.

We proceed to enquire whether A. E, I and O may all be inverted, and if so, by which process.

Inversion of A explained.

Let us see if A can be inverted by beginning with conversion. From S a P—All S is P (original proposition), we have P i S—Some P is S (converse), and P o S′—Some P is not not-S (obverted converse). This being an O proposition cannot be converted again. Thus S a P cannot be inverted if we begin with conversion. Let us see what happens if we begin with obversion. We have S a P—All S is P (original proposition); S e P′—No S is not-P (obverse); P′ e S—No not-P is S (contrapositive); P′ a S′—All not-P is not-S (obverted contrapositive); (by conversion) S′ i P′—Some not-S is not-P (obverted inverse or full inverse); (by further obversion) S′ o P—Some not-S is not P (inverse or partial inverse). The following is a concrete example: All virtuous men are happy (original proposition); No virtuous men are non-happy (obverse); No non-happy men are virtuous (contrapositive); All non-happy men are non-virtuous (obverted contrapositive); Some non-virtuous men are non-happy (obverted inverse or full inverse); Some non-virtuous men are not happy (inverse or partial inverse).

Inversion of E explained.

Let us now see what happens if we try to invert E by beginning with obversion. From S e P—No S is P (original proposition), we have S a P′—All S is not-P (obverse); P′ i S—Some not-P is S (contrapositive; P′ o S′—Some not-P is not not-S (obverted contrapositive). This being an O proposition cannot further be converted. So we cannot invert E if we begin with obversion. Let us therefore begin with conversion.

Then we have : S e P—No S is P (original proposition) ; P e S—No P is S (converse) ; P a S′—All P is not-S (obverted converse) ; S′ i P—Some not-S is P (inverse or partial inverse) ; S′ o P′—Some not-S is not not-P (obverted inverse or full inverse). The following is a concrete example : No cruel man is praiseworthy (original proposition) ; No praiseworthy man is cruel (converse) ; All praiseworthy men are non-cruel (obverted converse) ; Some non-cruel men are praiseworthy (inverse or partial inverse) ; Some non-cruel men are not other than those who are praiseworthy (obverted inverse or full inverse).

I and O propositions cannot be inverted.

I and O **cannot be inverted.** S i P if obverted becomes S o P′, but S o P′ cannot be converted, being an O proposition. Again S i P if converted becomes P i S, which when obverted becomes P o S′ ; but this again, being an O proposition, cannot be converted. So I cannot be inverted by either method. S o P, being an O proposition, cannot be converted. If we begin with obversion, it becomes S i P′, which being converted becomes P′ i S ; P′ i S obverts to P′ o S′, but this, being an O proposition, cannot be converted again. Thus O cannot be inverted at all.

In conclusion we may point out that the partial inverse of A being S′ o P, P is distributed in the conclusion, as it is the predicate of a negative proposition, though it is not distributed in the premise S a P. Thus the inversion of A apparently involves a flaw. But the inverse S′ o P is obtained by obversion and conver-

sion, which are valid processes of reasoning, and therefore the inversion of A need not be regarded as faulty. It should also be observed that both A and E invert to I and O. Accordingly, **inversion is always** per accidens or **by limitation.**

The following chart illustrates the various results obtained by the eductions discussed above :—

	A	I	E	O
Original proposition ..	S a P	S i P	S e P	S o P
Obverse	S e P′	S o P′	S a P′	S i P′
Converse	P i S	P i S	P e S	
Obverted converse ...	P o S′	P o S′	P a S′	
Partial contrapositive or contrapositive ...	P′e S		P′ i S	P′ i S
Full contrapositive or obverted contrapositive .. .	P′a S′		P′ o S′	P′ o S′
Partial inverse or inverse	S′ o P		S′ i P	
Full inverse or obverted inverse	S′ i P′		S′ o P′	

Other Eductions.

There are certain cases of immediate inference which cannot strictly be regarded as formal and cannot be expressed by symbols. Such inferences have been called

material immediate inferences as distinct from formal immediate inferences. We may consider these eductions now.

1. Inference by Added Determinants

In inference by added determinants we **add** the **same qualification** to both the **subject** and the **predicate** of a proposition and hold the result of our operation to be true, on the strength of the truth of the original proposition. According to Keynes, "Immediate inference by added determinants is a process of immediate inference which consists in limiting both the subject and the predicate of the original proposition by means of the same determinant." All P is Q, ∴ All AP is AQ. A negro is a fellow-creature; therefore a suffering negro is a suffering fellow-creature. A mango is a fruit; therefore a sweet mango is a sweet fruit. Leibniz expresses this inference by mathematical symbols: *e.g.* "If $A=B$ and $L=M$, then $A+L=B+M$." This is admissible, provided the sign (=) is not taken to signify equality but merely to denote the logical copula 'is,' and that the sign (+) simply implies a connexion of elements, not a mathematical addition of units. Keynes gives a proof of the formal validity of the inference, P is Q, ∴ AP is AQ. Thus: AP is a sub-class of P, but P is Q, ∴ AP is Q. But AP is also A; *i.e.* AP is both A and Q; *i.e.* AP is AQ. But the truth of the inference by added determinants, as it depends upon knowledge of facts, cannot properly be expressed by symbols, the truth of which holds good universally. We cannot pass from the proposition 'An ant is an

animal' to the conclusion that 'a large ant is a large animal.' Similarly we cannot conclude that 'a good teacher is a good man' from the premise that 'a teacher is a man.'

2. Immediate Inference by Complex Conception

In this kind of inference, "the **subject** and **predicate** of a given proposition are used to **qualify** in some way the **same term**, and thus **complex concepts** are formed that are made **subject** and **predicate** of a **new proposition.**" Keynes says, "Immediate inference by complex conception is a process of immediate inference which consists in employing the subject and the predicate of the original proposition as parts of a more complex conception." *E. g.*, Physics is a science, therefore physical treatises are scientific treatises. The dog is an animal, therefore the head of a dog is the head of an animal. In such inferences the subject and the predicate of the original proposition become adjectival in the conclusion, and form parts of more complex conceptions. Thus according to this mode of inference, if P is Q, it follows that whatever stands in a certain relation to P stands in the same relation to Q.

The validity of immediate inference by complex conception, like that of immediate inference by added determinants, does not rest upon purely formal grounds but upon knowledge of concrete facts. Thus from the proposition 'A cat is not a dog' we cannot pass to the conclusion that 'the owner of a cat is not the owner of a dog'. Similarly from the proposition 'Horses are animals' we cannot pass to the conclusion that 'the greater

number of horses is the greater number of animals.' Similarly we cannot say that since Protestants are Christians, the majority of Protestants are the majority of Christians.

Material eductions ought to be treated of in the special branch of logic known as the logic of relatives, and not in connection with formal immediate inferences. Material inference shows that "arguments are not all built on the principle of American watches, with interchangeable parts, so that terms from one may be transferred to another without interfering with the working of the inference ; and that the study of the inference, like the study of life, is largely a matter of examining types, though there are a certain number of common forms, which recur identically in diverse contents." Even in studying syllogism, which is the ideal type of formal inference, we cannot ignore the consideration of facts.

3. Immediate Inference by Converse Relation

This process is **analogous** to ordinary **conversion**, but properly belongs to the logic of relatives. In such an inference we pass from one statement of relation to another, that is, from the relation in which S stands to P to the relation in which P stands to S. Thus the subject and the predicate of the original proposition are transposed in the conclusion. Examples are : S is greater than P, therefore P is less than S. S is to the right of P, therefore P is to the left of S. S is to the east of P, therefore P is to the west of S. Alexander is the son of Philip, therefore Philip is the father of Alexander. Sita is the wife of Ram, therefore Ram is the husband

of Sita ; etc. Such inferences are not possible without knowledge of the implications of the relations involved. The distinction between formal and material inference is not absolute, but is made for the sake of convenience. The kinds of material inference illustrated above are treated of in the logic of relatives.

Two other usually recognised forms of immediate inference may be discussed briefly.

(a) Immediate Inference by Change of Relation is the process whereby we pass from a categorical to a hypothetical or a disjunctive proposition, or from a hypothetical to a disjunctive or a categorical proposition, or from a disjunctive to a hypothetical or categorical proposition. *E.g.*, All S is P (categorical), therefore If anything is S it is P (hypothetical). Rainy weather is wet weather, therefore If the weather is rainy, it is wet. Every S is either P or Q (disjunctive), therefore Any S that is not P is Q (categorical). Every man is either intelligent or non-intelligent, therefore Every man being not intelligent is non-intelligent. Every flower is either white or not-white (disjunctive), therefore If any flower is not white it is not-white (hypothetical). Similarly, If a man is honest he is trusted (hypothetical), therefore A man being honest is a man who is trusted (categorical). We need not give more examples. But a word of caution ought to be uttered, that in passing from a proposition of one relation to a proposition of another relation the meaning of the original proposition is very liable to suffer distortion, which care must be taken to avoid.

(b) Immediate Inference by Modal Consequence or Inference by Change of Modality is analogous to subaltern inference. In this variety of immediate inference we can pass from the validity of the apodeictic or necessary judgment to the validity of the assertoric, and from that to the validity of the problematic judgment, but not vice versa. On the other hand, from the invalidity of the problematic judgment we can pass to the invalidity of the apodeictic or necessary judgment, but not vice versa. Let us take the three propositions, S must be P (necessary), S is P (assertoric), S may be P (problematic). If the proposition S must be P is true, then the proposition S is P is true, and this being true, the proposition S may be P is also true. But if the proposition S may be P is true, we cannot conclude to the truth of S is P or of S must be P. Again if the proposition S may be P is false, then the propositions S is P and S must be P are also false. But if the proposition S must be P is false, then the other two propositions, S is P and S may be P, need not be false.

Eductions of Hypothetical and Disjunctive Propositions

Hypothetical and disjunctive propositions, before being made the basis of eductions, are better expressed in their denotative form and not in their abstract form. Thus hypothetical propositions, when they are reduced to the denotative form and are truly conditional, provide us with the four traditional forms, *viz.* If any S is M, then always that S is P (A); If any S is M, then never that S is P (E); If an S is M, then sometimes that S is P (I);

if an S is M, then sometimes not that S is P (O). In the case of disjunctive propositions, when the alternatives belong to the same subject, and are expressed in the denotative form, we may have the following four traditional forms: Every S is either P or Q (A); No S is either P or Q (E); Some S's are either P or Q (I); Some S's are neither P nor Q (O).

Eductions of hypothetical propositions.

Let us now see how the processes of eduction apply to hypothetical propositions. We first give a list of eductions from A:—

If any S is M then always that S is P (original proposition).

If any S is M, then never, that S is not P (obverse).

If an S is P, then sometimes, that S is M (converse).

If an S is P, then sometimes not, that S is not M (obv. conv.).

If any S is not P, then never, that S is M (contrapositive or partial contrapositive).

If any S is not P, then always, that S is not M (obv. contrapositive or full contrapositive).

If an S is not M, then sometimes not, that S is P (partial inverse or inverse).

If an S is not M, then sometimes, that S is not P (full inverse or obverted inverse).

Eductions from E:—

If any S is M, then never, that S is P (original proposition).

If any S is M, then always, that S is not P (obverse).

If any S is P, then never, that S is M (converse).

If any S is P, then always, that S is not M (obv. conv.).

If an S is not P, then sometimes, that S is M (contrapositive or partial contrapositive).

If an S is not P, then sometimes not, that S is not M (obv. contrapositive or full contrapositive).

If an S is not M, then sometimes, that S is P (partial inverse or inverse).

If an S is not M, then sometimes not, that S is not P (full inverse or obv. inv.).

Eductions from I :—

If an S is M, then sometimes, that S is P (orig. prop.).

If an S is M, then sometimes not, that S is not P (obverse).

If an S is P, then sometimes, that S is M (converse).

If an S is P, then sometimes not, that S is not M (obv. conv.).

Eductions from O :—

If an S is M, then sometimes not, that S is P (original proposition).

If an S is M, then sometimes, that S is not P (obverse).

If an S is not P, then sometimes, that S is M (contrapositive or partial contrapositive).

If an S is not P, then sometimes not, that S is not M (obv. contrapositive or full contrapositive).

Let us illustrate the eductions from A by a concrete example :—

If any man is virtuous he is happy (orig. prop.).

If any man is virtuous, then never is he not happy (obverse).

If a man is happy, he is sometimes virtuous (converse).

If a man is happy, he is sometimes not non-virtuous (obv. converse).

If any man is not happy, then never is he virtuous (partial contrapositive or contrapositive).

If any man is not happy he is non-virtuous (full contrapositive or obv. contrapositive).

If a man is not virtuous, he is sometimes not happy (partial inverse or inverse).

If a man is not virtuous, he is sometimes non-happy (full inverse or obv. inverse).

Eductions of disjunctive propositions.

We may now consider the eductions of disjunctive propositions. Eductions from A :—

Every S is either P or Q (original proposition).

No S is both P′ and Q′ (obverse).

Some things that are either P or Q are S (converse).

Some things that are either P or Q are not S′ (obv. conv.).

Nothing that is both P′ and Q′ is S (contrapositive or partial contrapositive).

Everything that is both P′ and Q′ is S′ (obv. contrapositive or full contrapositive).

Some S″s are neither P nor Q (partial inverse or inverse).

Some S″s are both P′ and Q′ (full inv. or obv. inv.).

The following is a concrete example :—

Every mango is either sweet or sour (orig. prop).

No mango is both non-sweet and non-sour (obverse).

Some things that are either sweet or sour are mangoes (converse).

Some things that are either sweet or sour are not other than mangoes (obv. converse).

Nothing that is both non-sweet and non-sour is a mango (contrapositive or partial contrapositive).

Everything that is both non-sweet and non-sour is not a mango (obv. contrap. or full contrap.).

Some things other than mangoes are neither sweet nor sour (partial inv. or inv.).

Some things other than mangoes are both non-sweet and non-sour (full inv. or obv. inv.).

We need not illustrate eductions from E, I and O in the case of disjunctive propositions, because they do not involve any new principle.

CHAPTER II

PURE SYLLOGISMS

Definition and Nature of Syllogism

Syllogism

Syllogism regarded as the most important form of deductive or formal inference.

the most important form of deductive inference. We have already discussed the different forms of immediate inference, and it is undeniable that the elements of inference involved therein are not very important. Some therefore have refused to regard immediate inference as inference proper. But we have pointed out that immediate inference involves some, even if only slight, movement of thought, and therefore it should be regarded as inference. Syllogism on the other hand has from the time of Aristotle been regarded as the ideal form of deductive or formal inference. No one has ever refused to call it inference, though some logicians, such as Mill and his followers, have questioned the capacity of syllogistic inference to establish valid conclusions. In a **syllogism** we draw a **conclusion** from **two premises.** not from one premise only as in immediate inference. Every syllogistic reasoning requires **three** and only three **propositions,** two of which, called **premises** or **implicants,** are **given**; and the other is called the **conclusion** or the **implicate.** Further, the syllogism requires three and only **three terms.** A syllogism may be regarded either as an **implication.** in which case the propositions are called the

implicants and the implicate, or it may be regarded as an **inference,** in which case the propositions are called the premises and the conclusion. Syllogism is one of the **demonstrative** forms of inference, for within its limits its validity cannot be questioned; that is, from the formal point of view the truth that it establishes on the basis of given truths has demonstrative certainty. The etymological meaning of the term syllogism is 'collecting together', and this meaning is significant inasmuch as the elements of a syllogism are thought together.

Syllogism a demonstrative form of inference.

Syllogism has been defined by Johnson as "an argument containing **two premises** and a **conclusion,** involving between them **three terms, each** of which occurs in **two** different propositions". According to Joseph, "A syllogism is actually an argument in which, from the given relation of two terms, *in the way of subject and predicate*, to the same third term, there follows necessarily a relation, *in the way of subject and predicate*, between those two terms themselves". If we take the syllogism All M is P, All S is M, therefore All S is P, we find that it consists of three propositions, of which the first two are the premises and the last one is the conclusion; and contains three terms, P, M and S. The term which is the **predicate** of the **conclusion** is called the **major term,** that which is the **subject** of the **conclusion** is called the **minor term,** and that which occurs in the **premises** but **not** in the **conclusion** is called the **middle term.** In the above example P **is** the major

Syllogism defined and its nature explained.

term, S the minor term and M the middle term. The proposition in which the **major term** occurs is called the **major premise**, and that in which the **minor term** occurs is called the **minor premise**; in the conclusion both the major and the minor term are present. The **major** and the **minor** term are called the **extremes**, because they are the extremities of the conclusion. The relation between M and P. M and S, and S and P is in the way of subject and predicate. In the premises, S and P are related to the third term M in the way of subject and predicate, and in the conclusion S and P are related in the same way.

Function of the middle term.

The **middle** term **mediates** the conclusion by **connecting** the **major** and the **minor** terms. According to Bosanquet, the middle term may be regarded as the "copula or grip which holds the conclusion together, made explicit and definitely stated." It was supposed by older logicians that the major term has the largest extent, the minor term the least extent, and the middle term an extent intermediate between the major and minor. In a syllogism in which all the propositions are universal affirmatives, *i.e.* A propositions (as in the above example), the extent of the major term is larger than that of the middle term, and that of the middle term is larger than that of the minor. But in a negative syllogism this does not hold good. Take the syllogism, No men are monkeys, John is a man, therefore John is not a monkey. Here we cannot say whether the extent of the major term is larger or smaller than, or equal to, that of the middle term. The **middle** term is so called because it **establishes a connec-**

tion between the major and the minor terms, and thus mediates the conclusion.

Order of propositions in a syllogism not important.

Usually the major premise is stated first and then the minor premise, and the conclusion at the end. But in actual reasoning this order may not be strictly followed, nor is the order of the syllogism important. We cannot agree with Jevons when he remarks that the cogency of a syllogistic argument becomes apparent if the minor premise is stated first. The usual order is satisfactory, inasmuch as the major premise is generally universal and provides the rule of which the minor premise is a particular application. Though the order of the propositions in a syllogism has no logical significance, it may be of some rhetorical importance. The older logicians stated the conclusion, which they called the question, first, and the premises, which they called the reason, afterwards. The order followed was—

Socrates is mortal (Question)

because

$$\left.\begin{array}{r}\text{All men are mortal}\\ \text{and Socrates is a man}\end{array}\right\}\ (\textbf{Reason}).$$

It was also customary to call the major premise the principle or proposition, the minor premise the reason or assumption, and the conclusion the deduction or collection.

Syllogism is mediate deductive inference, in which the conclusion can never be more general than the premises. The truth of the conclusion follows from the truth of the

premises. According to Welton and Monahan, "The whole force of a syllogism depends upon the necessity with which the inferred proposition follows from those given as data, and this necessity must be evident from the mere form of the argument." The conclusion of a syllogism, being derived from the premises, is not absolutely a new proposition. Though it follows necessarily from the premises, nevertheless it goes beyond them ; but in formal inference the element of necessity is more important than the element of novelty. Syllogistic inference is similar to the multiplication of large numbers, in which the product, though a new number, is implied by the data and follows from them. In formal inference, we do not question the validity of the premises, which are accepted as true, and our problem is to derive a conclusion from them in conformity with the rules of syllogism. If the premises are true materially, the conclusion also is materially true ; *e.g.*, All birds have wings, This sparrow is a bird, therefore This sparrow has wings. It may happen that the premises are materially false while the conclusion is materially true. But this is accidental, and is a case of mere coincidence, not of necessity. For example, in, All birds are rational, Men are birds, therefore Men are rational, the conclusion is materially true though the premises are materially false.

Syllogism a type of mediate inference, in which the conclusion is never more general than the premises.

In formal inference the element of necessity is more important than the element of novelty.

The **terms** of a syllogism are called its **remote matter,** the **propositions** its **proximate matter.** "The matter of a syllogism is given in its terms, which vary according to

the subject to which the argument refers. Its form consists in that relation of the terms by which they are united in two propositions necessitating a certain conclusion" (Welton).

The opposite of syllogism is antilogism, the propositions of which cannot be true together.

The opposite of syllogism is **Antilogism.** Whereas in syllogism all the three propositions must be consistent with one another and true together, in antilogism the **propositions cannot be true at once.** Johnson gives the following example of antilogism : 'All tactful persons sometimes lie ; Lord Grey is a tactful person ; Lord Grey never lies'. If the propositions 'Lord Grey is a tactful person' and 'Lord Grey never lies' are true, then the proposition 'All tactful persons sometimes lie' cannot be true. Again if the propositions 'All tactful persons sometimes lie' and 'Lord Grey is a tactful person' are true, then 'Lord Grey never lies' cannot be true. In the same way if the propositions 'All tactful persons sometimes lie' and 'Lord Grey never lies' are true, then 'Lord Grey is a tactful person' cannot be true. Thus in an antilogism the propositions are together incompatible.

We may in passing remark that **syllogism,** being a type of **formal** inference, can be represented by **symbols.**

Pure and mixed syllogisms contrasted.

Syllogisms may be either **pure** or **mixed. Pure** syllogisms are those in which the **propositions** are of the **same relation.** Thus in a pure categorical syllogism all the propositions are **categorical, in** a pure

hypothetical syllogism all the propositions are hypothetical, and in a pure disjunctive syllogism all the propositions are disjunctive. All pure syllogisms, whether categorical, disjunctive, or hypothetical, are subject to the same rules, and the distinction between them is not important. **Mixed** syllogisms are (1) **Mixed hypothetical,** also called hypothetico- categorical, (2) **Mixed disjunctive,** also called disjunctive-categorical, and (3) **Dilemma.** The different forms of mixed syllogism being subject to different rules, the distinction between them is important. In a **mixed hypothetical** syllogism, the **major** premise is **hypothetical** and the **minor** premise **categorical.** In a **mixed disjunctive** syllogism, the **major** premise is **disjunctive** while the **minor** premise is **categorical.** In a **dilemma,** the **major** premise is **hypothetical** and the **minor** premise is **disjunctive.** In this and the following chapter, we shall discuss the different forms of pure syllogisms, and shall devote a separate chapter to the consideration of the forms of mixed syllogism.

Pure Syllogisms:
1. Pure categorical ;
2. Pure hypothetical ;
3. Pure disjunctive.

Mixed Syllogisms:
1. Mixed hypothetical or hypothetico-categorical ;
2. Mixed disjunctive or disjunctive-categorical ;
3. Dilemma.

Axioms of Pure Syllogisms

We are now in a position to state and explain the axioms and general rules of syllogisms. We must not suppose that these axioms and general rules are presupposed in syllogistic inference; rather they are deductions from the nature of syllogistic reasoning. These principles are helpful, inasmuch as they enable us to test the correctness of different pieces of syllogistic reasoning. In syllogistic reasoning, we must remember, we are concerned not with the validity of premises and conclusion taken separately, but with the validity of the process of reasoning involved. Therefore it has been called the logic of consistency, as opposed to inductive logic, which is supposed to be the logic of truth. But we shall find in the course of our discussion that a good deal of induction rests upon formal grounds, though at the same time it depends upon observation of and experiment upon facts.

Axioms and rules of syllogism are deductions from the nature of syllogistic reasoning.

All reasonings, including syllogistic inference, depend upon the fundamental **laws of thought.** Every **affirmative categorical** syllogism rests upon the principle of **identity.** Take the syllogism, All organisms are mortal, Human beings are organisms, therefore Human beings are mortal. In this argument the middle term 'organisms' is identical with 'mortal' in one premise and with 'human beings' in the other, and hence we can establish a relation of identity

Affirmative categorical syllogisms rest on the principle of identity, negative ones on the principle of contradiction; pure hypothetical syllogisms require also the principle of sufficient reason.

between 'human beings' and 'mortal'. By 'identity' however we mean not absolute identity, but identity amidst diversity. **Negative categorical** syllogisms rest upon the principle of **contradiction.** In the argument, No negroes are white, Joseph is a negro, therefore Joseph is not white, the major premise asserts a separation between the middle term 'negroes' and the major term 'white,' while the minor premise asserts a relation of identity between the middle term and the minor term 'Joseph,' and therefore the conclusion asserts a relation of exclusion between the major term and the minor term. Thus in an affirmative syllogism the identity between the extremes is established through the medium of the middle term, while in a negative syllogism the relation of exclusion is established between the extremes, because the middle term in one of the premises excludes one of the extremes, while in the other premise it is connected with the other extreme by a relation of identity. A **pure hypothetical** syllogism depends as much upon the principle of **sufficient reason** as upon the principles of identity and contradiction. If any man is a philanthropist, he is loved by many ; If any man serves other men in a disinterested manner, he is a philanthropist ; therefore If any man serves other men in a disinterested manner, he is loved by many, is an example of a pure hypothetical syllogism which rests upon the principle of identity and that of sufficient reason.

Various **axioms** of syllogisms have been proposed by logicians. The most famous is the 'Dictum de omni et nullo' of Aristotle, which was supposed by him to be the ground of all syllogistic reasonings. But the **axioms** of

Axioms of categorical syllogisms given by Whately and Hamilton.

syllogisms proposed by philosophers are **not fundamental**, because they may be **deduced** from the fundamental **laws of thought**, such as the law of identity, the law of contradiction, etc. They are therefore to be regarded as **middle axioms** derived from the fundamental ones. Before explaining Aristotle's dictum we may state the axioms proposed by Whately and Hamilton. Whately gives the following two axioms, upon which, according to him, all syllogistic reasonings are based: (a) "If two terms agree with one and the same third, they agree with each other." (b) "If one term agrees and another disagrees with one and the same third, these two disagree with each other." Axiom (a) is supposed to be the basis of affirmative syllogisms, axiom (b) that of negative syllogisms. Hamilton gives the following axiom, which he calls "the supreme canon of categorical syllogisms": "In so far as two notions (notions proper and individuals) either both agree, or one agreeing the other does not agree, with a common third notion, in so far these notions do or do not agree with each other." These axioms of Whately and Hamilton are unassailable, but they cannot be regarded as supreme, for they are derived from the principles of identity and contradiction, of which they are developed statements. They should not be called canons (*i.e.* rules), but middle axioms. There is no essential difference between the axioms of Whately and that of Hamilton. The only difference is that Whately uses the terminology of nominalists while Hamilton uses that of conceptualists. Hamilton's single axiom sums up the two axioms of Whately.

The axiom given by Aristotle is known as the **Dictum de omni et nullo**,[1] which is rendered in English by Welton and Monahan as—'Whatever is distributively predicated, whether affirmatively or negatively, of any class, may be predicated in like manner of anything which can be asserted to belong to that class.' The empty scheme of this axiom is M—P, S—M, S—P. This axiom requires that the middle term should be the subject of the major premise and the predicate in the minor. It also requires that the major premise should be universal and the minor affirmative, and that the conclusion should be affirmative if the major premise is affirmative, and negative if the major premise is negative. Aristotle and his followers supposed that if a syllogistic reasoning does not appear in the form required by the dictum, it should be reduced to that form in order that its validity may be proved. We may point out that Aristotle's axiom, like those of Hamilton and Whately, is the development of the two fundamental laws of thought, *viz.* the law of identity and the law of contradiction, and should like them be regarded as a middle axiom. We shall find in the course of our discussion that a syllogistic reasoning may be valid even though it does not appear in the form required by the dictum.

Aristotle's dictum explained.

Mill finds fault with the dictum of Aristotle, which rests upon the class-inclusion theory of predication; that is, according to Aristotle the terms of every proposition are to be read in denotation. Mill argues that

1 De omni et nullo, literally='Concerning all and none'.

general terms are usually read in connotation, and denotation depends upon connotation. Therefore the dictum of Aristotle has to be modified. So according to Mill 'Nota notae' (lit., 'mark of a mark') is a better dictum. This may be rendered in English either as (1) "Whatever has any mark, has that which it is a mark of", or as (2) "Whatever is a mark of any mark, is a mark of that which this last is a mark of." The first reading applies well to such a syllogism as, All men are mortal, Socrates is a man, therefore Socrates is mortal. Here the major premise implies that the attributes of men are associated with those of mortal, the minor premise implies that Socrates possesses human attributes, and the conclusion implies that Socrates possesses the attributes of mortality. In other words Socrates has the mark man, and man has the mark mortal, and therefore Socrates has the mark mortal. The second reading of the dictum is suited to syllogisms in which both the premises are universal, *e.g.* All men are mortal, kings are men, therefore kings are mortal. In this case the major premise implies that the attributes of men are associated with the attribute mortality, the minor premise implies that the attributes of kings are associated with those of men, and the conclusion implies that the kingly attributes are associated with the attribute mortality. In other words, the attributes of kings are a mark of those of man, the attributes of man are a mark of the attribute mortality, and therefore the attributes of kings are a mark of the attribute mortality.

Explanation of the dictum Nota Notae, which is preferred by Mill.

General Rules or Canons of Categorical Syllogisms

Logicians generally recognise **six general rules of syllogism** and three **corollaries** from them. Of the six rules, two refer to the **nature** of the syllogism, two to the **distribution of terms** in it, and the remaining two to the **quality** of the **propositions** of the syllogism. We may deduce them all from the dictum of Aristotle explained above, by a slight generalisation of it. We may now explain these rules :—

I. A syllogism must contain **three** and only three **terms.**

II. A syllogism must consist of **three** and only three **propositions.**

The first two rules serve to define the syllogism.

These two rules serve to **define** the syllogism as a particular form of argument. They are **not** required to prove the **validity** of a syllogistic argument. They only state the **nature** and **structure** of the syllogism. We know that in syllogistic reasoning we establish a relation between two terms in the way of subject and predicate, because these terms are related to a common third term in the premises in the way of subject and predicate. So in a syllogism there can be only three terms. Similarly a syllogism requires that a conclusion should be drawn from two and only two premises, and therefore there cannot be more or less than three propositions in a syllogism. An argument may be valid if it contains more or less than three terms, or more or less than three propositions, but such an argument is not a

syllogism. Thus we have seen that an immediate inference consists of only two propositions, yet it is a valid form of reasoning. In the same way a train of syllogisms may be a valid form of reasoning, though it contains more than three terms and more than three propositions.

If any of the terms of a syllogism is ambiguous, then there is the Fallacy of Four Terms or Quaternio Terminorum.

If one of the terms of a syllogism is ambiguous, then it becomes equivalent to two terms, and strictly speaking the syllogism then presents us with four terms instead of three. If any term of a syllogism is ambiguous, it commits the **fallacy of four terms,** technically known as Quaternio Terminorum. Ambiguity of the middle term is frequent in syllogistic reasoning, and in this case the fallacy is commonly known as the fallacy of ambiguous middle. Similarly there may be the fallacy of ambiguous major or of ambiguous minor. The function of the middle term is to establish a relation between the two extremes, and if it is ambiguous, it fails to discharge its proper function. It is an axiom of logic that in the same argument the same term must be used in the same sense wherever it occurs. The violation of this axiom leads to fallacious reasoning. The following two arguments show how, through the ambiguity of the middle term of a syllogism, fallacy is committed. (1) De Morgan gives the following example: All criminal actions ought to be punished by law; Prosecutions for theft are criminal actions; therefore Prosecutions for theft ought to be punished by law. In this case the middle term 'criminal action' is ambiguous: in the major premise it means the committ-

ing of crime, while in the minor it means a form of legal procedure. (2) The second example to illustrate the fallacy of ambiguous middle is provided by Joseph: No vegetable has a heart, A good lettuce has a heart, therefore A good lettuce is not a vegetable. In this case the conclusion is fallacious, because the middle term 'heart' in the major premise means a particular kind of organ for maintaining the circulation of the blood, and the same term in the minor premise means the central part of the lettuce.

Deduction of the first two rules from the dictum of Aristotle.

We have stated that the general rules of syllogism can be deduced from the dictum of Aristotle. The dictum provides for three and only three terms and three and only three propositions. It says that whatever is predicated of a class can be predicated of anything belonging to that class. Thus whatever is predicated furnishes the major term, the class of which it is predicated is the middle term, and that which is included in the class is the minor term. Further, what is predicated of a class is stated by the major premise, that something belongs to the class is stated in the minor premise, and the conclusion predicates of that something what was originally predicated of the class.

The third rule explained.

III. The **middle term must be distributed in one at least of the premises.**—The function of the middle term is to connect the extremes, and therefore it must have identical reference in the two premises; but this can be secured only if it is distributed at least once. If the

middle term is not distributed in either of the premises, then the part of its extension to which the major term is related may be different from the part of its extension to which the minor term is related. If this happens, no connection can be established between the extremes. When the middle term is **not distributed** in **either** of the premises, we have the **fallacy of undistributed middle.** Thus we cannot draw any conclusion from the premises 'Some animals are herbivorous' and 'Some animals are carnivorous.' Similarly from the premises 'Some Indians are Hindus' and 'Some Indians are Moslems', no conclusion can be deduced. If a conclusion is drawn from such premises, it must be fallacious. This rule can also be proved by means of Euler's diagrams. But Euler's diagrams rest upon the assumption that the terms of a syllogism are always read in extension.

The violation of this rule leads to the fallacy of undistributed middle.

By a slight generalisation we can deduce this rule from the dictum of Aristotle. The dictum requires that in the major premise something must be predicated of a class distributively. This class is the middle term, and therefore the dictum provides that the middle term should be distributed in the major premise. By generalising this we get the rule that the middle term should be distributed once at least in the premises.

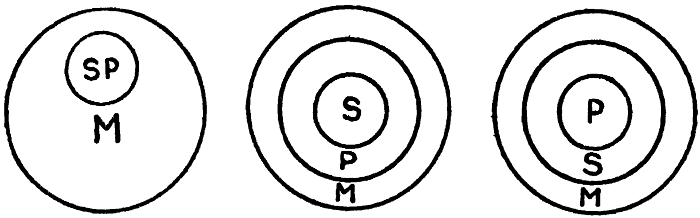
Deduction of the third rule from Aristotle's dictum.

As stated above, the rule that the middle term must be distributed in one of the premises may be illustrated by Euler's diagrams.

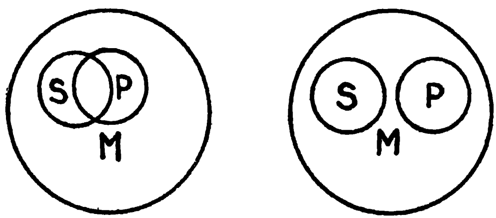
For, suppose we are given two premises such as

All P is M,
All S is M,

in neither of which is the middle term M distributed (since it is here in each case the predicate of an affirmative proposition). Then the relations between S, M, and P require the following five diagrams to represent them :



(i) (ii) (iii)



(iv) (v)

If we now look at the circles representing S and P, we find that all the five possible relationships between them are exemplified, and there can therefore be no conclusion, since any proposition stating a relation between S and P excludes at least one of the five relationships expressed by Euler's five diagrams.

Hence if the middle term is not distributed in either premise, there can be no conclusion.

IV. **No term** may be **distributed** in the **conclusion** which was **not distributed in one of the premises.**—

The fourth rule explained.

Syllogism requires that the conclusion should not be more general than the premises, and if a term is distributed in the conclusion without being distributed in the premise in which it occurs, there is a likelihood of the conclusion being more general than is warranted by the premises. If the **major** term is **distributed** in the **conclusion without being distributed** in the **major premise**, we have the fallacy of **illicit process of the major,** or **illicit major**.

Breach of this rule leads to the fallacy either of illicit process of the major or of illicit process of the minor term.

Thus if we draw the conclusion 'No horses are herbivorous' from the premises 'All cows are herbivorous' and 'No horses are cows', this fallacy occurs, because the major term is not distributed in the major premise, where it is the predicate of an affirmative proposition, while it is distributed in the conclusion, where it is the predicate of a negative proposition. If the **minor** term is distributed in the conclusion without being distributed in the minor premise, there is the fallacy of **illicit process of the minor,** or **illicit minor.** Thus if we draw the conclusion 'No mathematicians are capable of virtue' from the premises 'No politicians are capable of virtue' and 'Some mathematicians are politicians,' there is the fallacy of illicit process of the minor term, because it is not distributed in the minor premise, being the subject of a particular proposition, while it is distributed in the conclusion, being the subject of a universal proposition.

This rule also can be deduced from the dictum of

Aristotle by generalising the dictum. The dictum requires that predication should be made of anything belonging to a class, and this anything cannot be more definite in the conclusion than in the premise; that is, it cannot be distributed in the conclusion if it is not distributed in the minor premise. The predication made of 'this class' in the major premise can be made of 'this anything which belongs to this class' in the same way in the conclusion. Thus if the predicate of the major premise be undistributed, it should be undistributed also in the conclusion. Therefore by generalising the dictum we may obtain the canon that No term may be distributed in the conclusion which is not distributed in one of the premises.

Deduction of the fourth rule from Aristotle's dictum.

V. **From two negative premises nothing can be inferred.**—If the relation between the middle term and the extremes be denied in the premises, then the conclusion cannot establish a relation between the minor and the major term in the way of subject and predicate. In such a case the middle term cannot establish any connection whatever between the extremes. Only when one of the extremes is connected with the middle term can we, through that connection, infer its agreement with, or separation from, the other extreme. Thus from the premises 'No cows are carnivorous,' 'No horses are carnivorous', no conclusion can be drawn. Similarly from the premises 'No Hindu is a German', 'No Moslem is a German,' no conclusion can be inferred.

The fifth rule explained.

Jevons however argues that this rule cannot be accepted in its bare form. He gives the following example, in which a conclusion appears to be drawn from two negative premises : Whatever is not metallic is not capable of powerful magnetic influence ; carbon is not metallic ; therefore carbon is not capable of powerful magnetic influence. Here the conclusion is valid and it follows from the premises. We may give another similar example. From the premises, Whoever is not honest is not happy, Politicians are not honest, we can draw the conclusion, Politicians are not happy. But if we express these arguments symbolically, we find that they are in the form No not-M is P, No S is M, therefore No S is P. Here we have four terms instead of three. But by obverting the minor premise we obtain a syllogism in the form No not-M is P, All S is not-M, therefore No S is P. This argument is valid, and one of its premises is negative and the other affirmative. Thus we find that the argument given by Jevons does not really start from two negative premises. Again from the premises, John is not over 20 years of age, John is not under 20 years of age, we may draw the conclusion that John is 20 years of age. But this is not an example of syllogistic reasoning. The conclusion in this case follows from a disjunctive major premise, *viz.* that John is either over 20 years of age or under 20 years of age or 20 years of age, but he is neither over nor under 20 years of age, therefore he is 20 years of age.

The contention of Jevons that sometimes from two negative premises a conclusion can be inferred does not hold good.

The fifth rule, like the previous ones, can be deduced

from the dictum of Aristotle, since the dictum provides that the minor premise should state that something belongs to a class, and this can be expressed only by an affirmative proposition. It also provides that the major premise may be either affirmative or negative. Therefore by generalising the dictum we arrive at the rule that from two negative premises no conclusion can be inferred.

Deduction of the fifth rule from Aristotle's dictum.

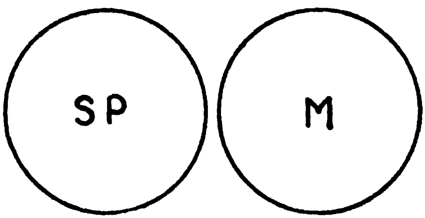
This rule also, like rule III, may be illustrated by Euler's diagrams.

Suppose we have given as premises two E propositions—

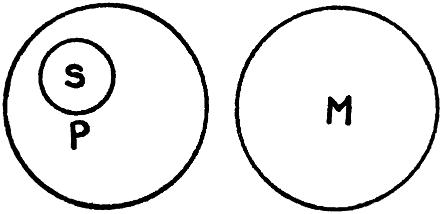
No P is M,

No S is M ;

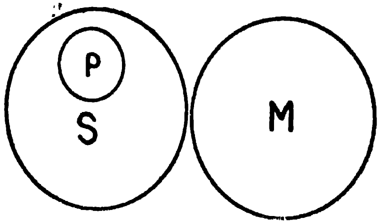
then the relationships between S, M and P are represented by the following five diagrams :—



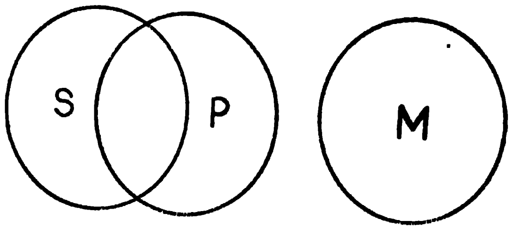
(i)



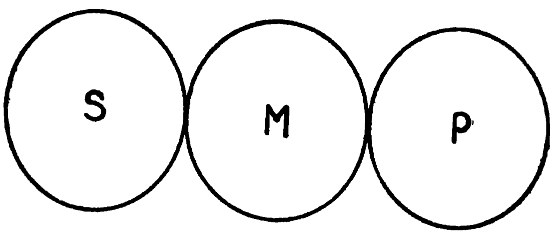
(ii)



(iii)



(iv)



(v)

If we now consider the circles representing S and P, we find that all five possible relationships between them are shown, and there can therefore be no conclusion, since any proposition stating a relation between S and P excludes at least one of the five relationships expressed by Euler's five diagrams.

Similarly it may be shown that no other possible pair of negative premises, whether EO or OO, can yield a

conclusion as to the relation between S and P, whatever be the order in which the terms are arranged.

VI. If **one premise is negative,** the **conclusion** must be **negative,** and to prove a negative conclusion, one of the premises must be negative.—If the middle term is in agreement with one of the extremes and is separated from the other, then the only relation which we can infer between the extremes is one of negation. Thus if M is in disagreement with P while it is in agreement with S, we can only infer that S and P are in disagreement. Thus if the premises be 'A is not equal to B', 'C is equal to B', then the legitimate conclusion is that C is not equal to A. From the premises 'No sheep are carnivorous', 'All my domestic animals are sheep', we can infer only a negative conclusion, *viz.* 'None of my domestic animals is carnivorous.' In the same way, if the conclusion asserts that the major term and the minor term are in disagreement, that is, a relation of negation exists between them, then such a relation can be inferred only when in the premises one of the extremes is in agreement and the other in disagreement with the middle term. If we want to establish the conclusion 'No men are perfect,' we can do so if the premises are 'No animals are perfect' and 'All men are animals'.

The sixth rule explained.

This rule also can be deduced from the dictum of Aristotle. The dictum provides that the major premise may be either affirmative or negative. It also provides that if the major premise is affirmative, the conclusion must be affirmative, and if it is

Deduction of the sixth rule from Aristotle's dictum.

negative, the conclusion must be negative. Further it requires that the minor premise must be affirmative. So by generalising the dictum we obtain the rule that if one of the premises is negative, the conclusion must be negative, and if the conclusion is negative, one of the premises must be negative.

When both the premises are affirmative, the conclusion must be affirmative, and to prove an affirmative conclusion the premises must be affirmative.

From the above rules it naturally follows that when the premises are both affirmative, the conclusion must be affirmative, and to prove an affirmative conclusion both the premises must be affirmative. We have already seen that from two negative propositions nothing can be inferred, that when one of the premises is negative, the conclusion must be negative, and that when the conclusion is negative, one of the premises must be negative. So only when both the premises are affirmative can the conclusion be affirmative, and when the conclusion is affirmative, both the premises must be affirmative. Aristotle's dictum also provides this in substance. It requires that the minor premise must be affirmative, and that when the major premise is affirmative the conclusion must be affirmative, but when the major premise is negative, the conclusion must be negative. This means that when both the premises are affirmative the conclusion is affirmative, and when one of them is negative the conclusion is negative. We may also point out that when both the premises are affirmative, they assert that the middle term is in agreement with both the extremes. When the premises assert such agreement between the extremes and the

middle term, the only conclusion we can infer is that the extremes are in agreement with each other. When A and B both agree with C we can only infer that they are in agreement with each other. Thus when the premises are All violets are white, This flower is a violet, the legitimate conclusion is that this flower is white. If X and Y are both equal to Z, then X and Y are equal to each other. Again when the conclusion asserts that there is agreement between the extremes, the premises from which it is inferred must also assert that each of the extremes is in agreement with the middle term. Thus if the conclusion is A is B, the premises must be affirmative, *e.g.* C is B and A is C.

The above rules are more or less independent of one another, but the remaining three rules follow from them and are of the nature of **corollaries.**

Seventh rule explained.

VII. **From two particular premises nothing can be inferred.**—If both the premises are particular, then the possible combinations are I I, OO, IO, OI. If the premises are I I, then they do not distribute any of the terms. But rule III requires the middle term to be distributed at least once in the premises. If the premises are OO, then nothing can be inferred from them, according to rule V, which states that from two negative premises nothing can be inferred. If the premises are either IO or OI, then any conclusion must be negative, according to rule VI, which states that if one of the premises is negative, the conclusion must be negative. If the conclusion is negative, then its predicate P, which is the major term, is distri-

buted in the conclusion. But the fourth rule requires that if a term is distributed in the conclusion, it must be distributed in one of the premises. So if we draw a conclusion from IO or OI, it is necessary to distribute the major term in the major premise. Further, according to rule III, the middle term must be distributed once in the premises. Thus two terms must be distributed in the premises, *viz.* the major term and the middle term, to warrant a conclusion; but IO or OI distribute between them only one term. If this term is the major term, then there is the fallacy of undistributed middle; if it is the middle term, then the major term remains undistributed in the premises. Thus if from IO or OI any conclusion is inferred, there is the fallacy either of undistributed middle or of illicit process of the major term. Thus it is proved that from two particular premises nothing can be inferred.

Rule VIII explained.

VIII. If **one premise** is **particular,** the **conclusion** must be **particular.**—If one of the premises is particular and the other universal, the possible combinations are AI, AO, IA, OA, EI, EO, IE, OE. Of these EO and OE yield no conclusion because both the premises are negative. AI and IA each distribute only one term, which according to rule III must be the middle term. So neither the major nor the minor term can be distributed in these combinations if the middle term is distributed. Therefore, if the conclusion is universal, the minor term is distributed in the conclusion without being distributed in the minor premise, involving the fallacy of illicit minor. AO, OA, EI, IE each distribute two terms. In each of these

combinations, one of the premises being negative the conclusion must be negative ; that is, in the conclusion, P, the major term, must be distributed. Therefore in the premises two terms must be distributed, *viz.* the major term and the middle term. If the conclusion is universal, then the minor term is distributed in the conclusion, and must therefore be distributed in the premises. But each of the above combinations distributes only two terms, and these, as we have pointed out, should be the major and the middle term. Therefore in these combinations there is no room to distribute the minor term in the premises. Therefore the minor term cannot be distributed in the conclusion ; that is, the conclusion must be particular. Thus if one of the premises is particular, the conclusion must be particular.

Rule IX explained.

IX. From a **particular major** and a **negative minor, nothing can be inferred.**—If the minor premise is negative the major must be affirmative, according to rule V. Now if the major premise is particular as well, the major term is not distributed there. But since one of the premises is negative, the conclusion must be negative, necessitating the distribution of the major term in the conclusion, where it is the predicate of a negative proposition. Thus if a conclusion is drawn from a particular major premise and a negative minor premise, there is the fallacy of illicit major. Thus if the major premise is particular and the minor negative, there cannot be any conclusion.

A As we have already pointed out, rules VII, VIII and IX are mere corollaries from the other six rules. Rules I

and II are purely descriptive, that is, they state what the elements of a syllogism are, and are not rules for proving the validity of a syllogistic reasoning. So the main rules are four in number, *viz.* two rules for distribution—rules III and IV, which state that the middle term must be distributed once at least in the premises, and that no term should be distributed in the conclusion which was not distributed in the premises; and two rules of quality—rules V and VI, which provide that from two negative premises nothing can be inferred, that if a premise is negative the conclusion must be negative, and that a negative conclusion necessitates a negative premise. We may also remark that rules IV and V may be shown to be involved in rule III, and rule VI in rule V; but we need not enter into intricacies to show how the four main rules imply one another.

Rules III, IV, V and VI may be regarded as fundamental.

Welton states that each of the two following pairs of rules may be regarded as fundamental, *viz.*—

(1) (*a*) The middle term must be distributed in one, at least, of the premises;

(*b*) To prove a negative conclusion requires a negative premise.

(2) (*a*) No term may be distributed in the conclusion which is not distributed in a premise;

(*b*) To prove a negative conclusion requires a negative premise.

According to Bain, when a conclusion is inferred from two singular propositions the argument is not syllogistic.

Some remarks on the view of Bain and Jevons regarding syllogism.

In such a case we have a compound proposition. Thus such an argument as 'Plato is the author of the *Republic*, Plato is an Athenian, therefore an Athenian is the author of the *Republic*,' is not really an argument but a compound proposition, *viz.* 'Plato, who is the author of the *Republic*, is an Athenian'. But if this is allowed, then such an argument as All men are mortal, All men are rational, therefore Some rational beings are mortal, may also be regarded as a compound proposition, *viz.* Men, who are mortal, are rational. Jevons holds that in the conclusion of a syllogism all the terms ought to be stated. Thus from the premises Potassium floats on water, Potassium is a metal, he draws the conclusion, Potassium metal is potassium floating on water. But this view cannot be accepted. The conclusion of a syllogistic reasoning must be a new proposition. Though it follows from the premises, it is not a mere summary of them. Further, in every reasoning there must be some movement of thought, but this is not apparent in the interpretation of syllogism given by Jevons. Further, the middle term should occur only in the premises, and not in the conclusion, in a syllogistic reasoning. But Jevons's interpretation does not satisfy this condition. His view that syllogistic reasoning is nothing but substitution cannot, therefore, be conceded.

We have seen that traditional logic holds that hypothetical propositions admit of fourfold division into A, E, I and O, and these distinctions are clearly evident when hypothetical propositions are expressed in the conditional

Syllogistic rules apply to pure hypothetical and pure disjunctive syllogisms.

or denotative form. If this is allowed, then the rules of syllogism given above apply not only to categorical but also to pure hypothetical syllogisms. Similarly, in the denotative form disjunctive propositions admit of the traditional fourfold division, as we have previously shown ; but since, properly speaking, disjunctive propositions cannot be negative, rules V and VI, which apply to syllogisms with negative propositions, do not apply to pure disjunctive syllogisms, but the remaining rules are applicable. A disjunctive proposition distributes its middle term only when one of the alternatives is a negative term. We shall show later how the rules of syllogism given above apply to pure hypothetical and pure disjunctive syllogisms.

Figures and Moods

Syllogisms are distinguished in **figure** according to the **position of the middle term.** "By the figure of a syllogism," says Keynes, "is meant the position of the terms in the premises." Four figures are recognised by logicians. Aristotle however did not recognise the fourth figure, which was first recognised by Galen, the famous physician, and is therefore named the Galenian figure. In the **first figure** the **middle term** is the **subject** in the **major** premise and **predicate** in the **minor.** The form of this figure is therefore M—P, S—M, S—P. In the **second figure** the **middle term** is the **predicate** in **both the premises,** and its form is P—M, S—M, S—P. In the **third figure** the **middle term** is the **subject in both**

Figure defined and its forms stated.

the premises, and its form is M—P, M—S, S—P. The **fourth figure** is the reverse of the first figure; in it the **middle term** is the **predicate** of the **major** premise and **subject** in the **minor.** The form of this figure is P—M, M—S, S—P.

Mood defined; number of possible moods.

"By the **mood** of a syllogism is meant the **quantity** and **quality** of the **premises** and **conclusion**" (Keynes). According to Joseph, "Syllogisms are distinguished in mood according to the quantity and quality of the propositions composing them." In each figure 16 combinations of premises are possible according to quality and quantity, *viz*. AA, AE, AI, AO, EA, EE, EI, EO, IA, IE, II, IO, OA, OE, OI, OO, and each of these yields four possible moods. Thus from AA we may have the moods AAA, AAE, AAI, AAO; similarly from AE we may have four possible moods, and so on. Thus in each figure 64 moods are possible. Therefore in the four figures together the number of possible moods is $64 \times 4 = 256$. But we shall find that of these only **24 moods** are **valid,** and others will be found wanting when tested. A mood which is valid in one figure may be invalid in another figure. Thus AAA (All M is P, All S is M, ∴ All S is P) is a valid mood in the first figure. But AAA in the second figure (All P is M, All S is M, ∴ All S is P) is not a valid mood, since it involves the fallacy of undistributed middle.

Every figure has its special rules. The **special rules** of the **first figure** are: (1) the **minor premise** must be

The special rules of the first figure stated and proved. They follow from the dictum of Aristotle.

affirmative; (2) the **major premise** must be **universal.** We remember that the form of the first figure is M—P, S—M, S—P. Now if the minor premise is not affirmative, that is, if it is negative, then the major premise must be affirmative (rule V), in which case the major term, as the predicate of an affirmative proposition, will not be distributed in the major premise. But again, if the minor premise is negative, the conclusion must be negative (rule VI), and in it the major term, which is its predicate, must therefore be distributed. Thus if the minor premise is negative, there is the fallacy of illicit process of the major term. Therefore the minor premise must be affirmative. If the minor premise is affirmative, then the middle term, which is the predicate of the minor premise, is not distributed there. It must therefore (by rule III) be distributed in the major premise, of which it is the subject. And to distribute its subject the major premise must be universal.

We may point out that these special rules of the first figure can be deduced from the dictum of Aristotle, which is the basis of the first figure. It provides that whatever is predicated of a class distributively, whether affirmatively or negatively, may be predicated in like manner of anything belonging to that class. Thus according to the dictum the major premise must make a predication about a class, and therefore it must be universal. The minor premise states that something belongs to that class, and therefore must be affirmative.

Let us now determine the **valid moods** of the **first**

figure with the help of the general rules and the special rules of the figure. We have seen that according to quantity and quality sixteen combinations of the premises are possible, *viz.* AA, AE, AI, AO, EA, EE, EI, EO, IA, IE, II, IO, OA, OE, OI, OO. Of these EE, EO, OE, OO yield no conclusion, according to the general rule V; II, IO, OI yield no conclusion, according to the general rule VII; and IE yields no conclusion, according to the general rule IX. Of the remaining combinations, AE, AO, IA, OA are not allowed by the special rules of the first figure. Thus AA, AI, EA, EI alone can yield conclusions in the first figure. From AA we may draw two conclusions, *viz.* A and I. Thus from All M is P, All S is M we may draw the conclusions All S is P and Some S is P. From AI we can draw the conclusion I. Thus from All M is P, Some S is M we obtain the conclusion Some S is P. From EA we may have E and O as the conclusions. The premises No M is P, All S is M may yield the conclusions No S is P and Some S is not P. EI yields the conclusion O. From the premises No M is P, Some S is M we may draw the conclusion Some S is not P. Thus the **valid** moods of the **first** figure are AAA, AAI, AII, EAE, EAO and EIO. Of these, AAI and EAO are called the **weakened** syllogisms of the first figure, because in these cases the premises allow A and E as conclusions respectively. These weakened moods are also called **subaltern** moods, since the conclusions in these cases are subalterns of A and E, which are valid conclusions. So only four moods being fundamental and independent in this figure, some

The valid moods of the first figure determined.

logicians recognise only these four instead of six; *viz.* AAA, EAE, AII, EIO. These are technically known as Barbara, Celarent, Darii and Ferio respectively. In each of these mnemonic words the vowels represent the propositions of the syllogism. We may now illustrate the four main moods of the first figure by concrete examples :—

1. AAA (Barbara)—All lions are tawny, This animal is a lion, therefore This animal is tawny.

2. EAE (Celarent)—No roses are green, All the flowers on my table are roses, therefore None of the flowers on my table is green.

3. AII (Darii)—All sheep are herbivorous, Some animals are sheep, therefore Some animals are herbivorous.

4. EIO (Ferio)—No negroes are white, Some Africans are negroes, therefore Some Africans are not white.

We may now proceed to discover the valid moods of the **second figure.** We must first state the **special rules** of the **second figure.** These are : (1) **one premise must be negative** ; (2) the **major premise** must be **universal.** In the second figure, (whose form is P—M, S—M, S—P), the middle term is the predicate in both the premises, and so, if neither of them is negative, the middle term will remain undistributed, which is contrary to the general rule III. Therefore one of the premises must be negative in the second figure. Now if one of the premises is negative, then the conclusion will be negative according to the general rule VI, and therefore the major

Special rules of the second figure stated and proved.

term must be distributed in the conclusion. Therefore it must also be distributed in the major premise (general rule IV). Now the major term is the subject of the major premise in the second figure, and can therefore be distributed only if the premise is universal.

Determination of the valid moods of this figure.

Which are the valid moods of the second figure? We have already found, while discussing the valid moods of the first figure, that of the sixteen possible combinations of premises, eight may be rejected at once as incapable of yielding any conclusion, according to the general rules. The remaining eight are AA, AE, AI, AO, EA, EI, IA, OA. Applying the special rules of the second figure to these eight combinations, we find that only four of them can yield valid conclusions in this figure, *viz*. AE, AO, EA, EI. From AE—All P is M, No S is M, we may have two conclusions, E and O, *viz*. No S is P and Some S is not P. Similarly from the combination EA—No P is M, All S is M, we may have two conclusions, E and O, *viz*. No S is P and Some S is not P. From the combination EI—No P is M, Some S is M we may draw the conclusion Some S is not P. Similarly the combination AO—All P is M, Some S is not M, yields the conclusion Some S is not P. Thus the **valid** moods of the second figure are six in number, *viz*., AEE, AEO, EAE, EAO, EIO and AOO. In this figure, as in the first, there are two **weakened** syllogisms, *viz*. AEO and EAO, because in each case the premises warrant an E proposition as the conclusion, and also because the truth of O follows from the truth of E, O being the subaltern of E. These two weakened syllogisms of this figure are

also called **subaltern** moods. The following mnemonics stand for the four fundamental valid moods of this figure :—Cesare, Camestres, Festino, Baroco. The vowels of these words again stand for the propositions of the syllogism.

Just as the special rules of the first figure may be deduced from the dictum of Aristotle, the special rules of the second figure are supposed to rest upon the **'dictum de diverso'**, which may be rendered in English as—"If a certain attribute can be predicated (affirmatively or negatively) of every member of a class, any subject, of which it cannot be so predicated, does not belong to the class" (Mansel). This dictum says that an attribute is predicated of a class, therefore it provides that one of the premises must be universal. It also says that there is to be a subject of which this attribute cannot be predicated. The dictum therefore requires that one of the premises is to be negative.

The special rules of the second figure can be deduced from the dictum de diverso.

We may now illustrate the main valid moods of this figure by concrete examples :—

1. EAE (Cesare)—No featherless animals are birds, All swans are birds, therefore No swans are featherless animals.

2. AEE (Camestres)—All doves are meek, No crows are meek, ∴ No crows are doves.

3. EIO (Festino)—No scarlet flowers are sweet-scented, Some flowers are sweet-scented, ∴ Some flowers are not scarlet flowers.

4. AOO (Baroco)—All warlike men are spirited, Some Indians are not spirited, ∴ Some Indians are not warlike men.

We may now state the **special rules of the third figure.** We must remember that the form of this figure is M—P, M—S, S—P. The special rules of this figure are: (1) the **minor premise** must be **affirmative;** (2) the **conclusion** must be **particular.** If the minor premise is negative, the major premise must be affirmative (rule V), in which case the major term, which is the predicate in the major premise in this figure, cannot be distributed in the major premise. But if the minor premise is negative, the conclusion must be negative (rule VI), and the major term, as its predicate, must be distributed there. But according to rule IV, no term can be distributed in the conclusion without being distributed in the premises. Therefore the minor premise cannot be negative in this figure, that is, it must be affirmative. Further, if the minor premise is affirmative, the minor term is not distributed in the minor premise, of which in this figure it is the predicate. It cannot therefore be distributed in the conclusion (rule IV). Therefore the conclusion must be particular.

The special rules of the third figure stated and proved.

Let us now see which of the eight combinations of premises, AA, AE, AI, AO, EA, EI, IA, OA, can yield valid conclusions in the third figure. In this figure, AE and AO cannot yield any conclusion, because in each case the minor premise is negative. AA—All M is P, All M is

Valid moods of the third figure.

S, yields the conclusion Some S is P. AI – All M is P, Some M is S, also yields the conclusion Some S is P. From EA – No M is P, All M is S, we may draw the conclusion Some S is not P. EI – No M is P, Some M is S, yields the conclusion Some S is not P. IA – Some M is P, All M is S, yields the conclusion Some S is P. OA – Some M is not P, All M is S, yields the conclusion Some S is P. The **valid moods** of the **third figure** are therefore AAI, AII, EAO, EIO, IAI and OAO. There is no weakened syllogism or subaltern mood in this figure, since it allows only particular conclusions. The mnemonics that represent the valid moods of this figure are—Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison. Here also the vowels of each of the mnemonics represent the propositions of the syllogism.

The syllogisms of the third figure are said to be based on the dictum de exemplo.

The special rules of the third figure are supposed to be based upon the **dictum de exemplo,** which is rendered into English as—"If anything which is stated to belong to a certain class is affirmed to possess, or to be devoid of, certain attributes, then those attributes may be predicated in like manner of some members of that class" (Welton). This dictum requires that the minor premise should state that something belongs to a class, and therefore it must be affirmative; and also that the conclusion should state that some attributes are to be predicated of some members of that class, which necessitates the conclusion's being particular.

Let us now illustrate the valid moods of this figure by concrete examples :—

1. AAI (Darapti)—Socrates is a Greek, Socrates is wise, ∴ Some wise men are Greeks.

2. IAI (Disamis)—Some men are honest, All men are mortal, ∴ Some mortals are honest.

3. AII (Datisi)—All birds are winged, Some birds are musical, ∴ Some musical beings are birds.

4. EAO (Felapton)—No negroes are white, All negroes are uncivilised, ∴ Some uncivilised men are not white.

5. OAO (Bocardo)—Some flowers are not red, All flowers are short-lived, ∴ Some short-lived objects are not red.

6. EIO (Ferison)—No men are perfect, Some men are virtuous, ∴ Some virtuous beings are not perfect.

The special rules of the fourth figure stated and explained.

We may now determine the **valid moods** of the **fourth figure.** We must remember that the form of the fourth figure is P—M, M—S, S—P. The **special rules** of the fourth figure are : (1) **if** the **major** is **affirmative** the **minor** must be **universal ;** (2) **if either** premise is **negative,** the **major** must be **universal ;** (3) **if** the **minor is affirmative,** the **conclusion** must be **particular.** If the major premise is affirmative, then the middle term, which is the predicate of the major premise, will remain undistributed in that premise. It must therefore be distributed in the minor premise (general rule III), and as it is the subject of that premise it can be distributed

there only if the minor premise is universal. Thus the first of the above rules is proved. Again, if one of the premises be negative, then the conclusion will be negative (general rule VI), and the major term will therefore be distributed in the conclusion. It must therefore be distributed in the major premise. Now, in the fourth figure the major term is the subject of the major premise, and can be distributed in that premise only if it is universal. Therefore if one of the premises is negative, the major premise must be universal. Thus the second special rule is proved. Again, if the minor premise be affirmative, the minor term, which is the predicate of that premise in the fourth figure, is not distributed in the premise. Therefore it cannot be distributed in the conclusion (general rule IV). Hence the conclusion must be particular, otherwise the minor term, which is the subject of the conclusion, will be distributed there though it is not distributed in the minor premise. Thus the third special rule is proved.

Valid moods of the fourth figure.

Let us now see which of the eight combinations of premises, AA, AE, AI, AO, EA, EI, IA, OA, yield valid conclusions in the fourth figure. In the fourth figure, AI, AO and OA cannot yield valid conclusions, according to the special rules of this figure. AI and AO are not allowed by the first special rule. OA is not allowed by the second special rule. From AA—All P is M, All M S, we may have the conclusion Some S is P. From AE—All P is M, No M is S, we may have two conclusions, *viz.*, No S is P and Some S is not P. From IA—Some P is M,

All M is S we may draw the conclusion Some S is P. From EA—No P is M, All M is S, we can draw the conclusion Some S is not P. EI—No P is M, Some M is S, yields the conclusion Some S is not P. Thus the **valid** moods of the **fourth** figure are AAI, AEE, AEO, IAI, EAO, and EIO. Of these moods AEO is a **weakened** syllogism, because the premises allow an E proposition as the conclusion, and the truth of O, which is the subaltern of E, follows from the truth of E. This mood is also called the **subaltern** mood of the fourth figure. If we leave out the subaltern mood, the other valid moods are represented by the mnemonics Bramantip, Camenes, Dimaris, Fesapo, Fresison. The vowels of each of these mnemonics represent the propositions of the syllogisms, as in the other figures.

The special rules of this figure are supposed to follow from Lambert's dictum de reciproco.

The special rules of this figure are supposed to follow from Lambert's **'dictum de reciproco'**, which is rendered in English as—"Whatever has a predicate affirmed, or universally denied, of it, may itself be predicated, particularly and with like quality, of anything which is affirmed of that predicate; and whatever has a predicate universally affirmed of it may itself be universally denied of anything which is universally denied of that predicate." The following are concrete examples of the main valid moods of this figure :—

1. AAI (Bramantip)—All philosophers are men, All men are mortal, ∴ Some mortals are philosophers.

2. AEE (Camenes)—All roses are sweet-scented, No

sweet-scented things are unpleasant, ∴ No unpleasant things are roses.

3. IAI (Dimaris)—Some men are virtuous, All that are virtuous are happy, ∴ Some happy beings are men.

4. EAO (Fesapo)—No politicians are honest, All honest men are trusted, ∴ Some that are trusted are not politicians.

5. EIO (Fresison)—No sheep are carnivorous, Some that are carnivorous are ferocious, ∴ Some that are ferocious are not sheep.

Summary of the valid moods of all the figures.

Our examination thus shows that six moods are valid in each figure. In the first figure the valid moods are AAA, AAI, EAE, EAO, AII, EIO. The valid moods of the second figure are EAE. EAO, AEE, AEO, EIO, AOO. Those of the third figure are AAI, IAI, AII, EAO, OAO, EIO. Those of the fourth figure are AAI, AEE, AEO, IAI, EAO and EIO. The above list shows that A can be proved only in one mood and in one figure, *viz*. in AAA in the first figure. E can be proved in four moods, and in every figure except the third. The moods that prove E are—EAE (fig. 1), EAE, AEE (fig. 2), AEE (fig. 4). I can be proved in seven moods, and in every figure except the second. The moods that prove I are AII and AAI in fig. 1, AAI, IAI and AII in fig. 3, AAI and IAI in fig. 4. O can be proved in twelve moods, and in every figure. The moods that prove O are EAO and EIO in fig. 1; EAO, AEO, EIO and AOO in fig. 2; EAO, OAO and EIO in fig. 3; AEO, EAO and EIO in fig. 4. We have also seen that of the 24 valid moods, five are subaltern

moods, *viz.*, AAI, EAO in fig. 1; EAO, AEO in fig. 2; and AEO in fig. 4. Many logicians do not mention these moods when they draw up a list of valid moods of all the figures. If we ignore these weakened syllogisms or subaltern moods, the fundamental valid moods in all the figures are 19 in number. These are all mentioned in the following mnemonic lines:

*Barbara, Celarent, Darii, Ferio*que prioris:[1]
Cesare, Camestres, Festino, Baroco secundae:[1]
Tertia[1] *Darapti, Disamis, Datisi, Felapton,*
Bocardo, Ferison habet[1]: quarta insuper addit[1]
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

Strengthened and weakened syllogism explained.

"If in a syllogism the same conclusion can still be obtained although for one of the premises we substitute its subaltern, the syllogism is said to be a **strengthened** syllogism" (Keynes). In a strengthened syllogism one of the premises is unnecessarily strengthened, that is, it can be replaced by its subaltern without affecting the conclusion. Thus Darapti—All M is P, All M is S, ∴ Some S is P, is a strengthened syllogism, because the same conclusion can be obtained even if one of the premises is replaced by its subaltern. In three moods, *viz.* AAI in fig. 3, and EAO in figs. 3 and 4, the middle term is twice distributed, which is not demanded by any

[1] Prioris='of the first' (figure); secundae='of the second' (figure); tertia...habet='the third (figure) includes' (the moods mentioned in between the two words); 'quarta insuper addit'='the fourth (figure) adds over and above'.

of the rules of the syllogism. Similarly Bramantip—All P is M, All M is S, ∴ Some S is P—is a strengthened syllogism. In this case the major term is undistributed in the conclusion, while it is distributed in the major premise; and hence for the major premise we may substitute its subaltern, which is an I proposition, and still obtain the same conclusion. Every syllogism with universal premises and a particular conclusion is strengthened, with the single exception of AEO in the fourth figure. Therefore all the weakened syllogisms except AEO are at the same time strengthened ones, and in each of these cases a particular conclusion follows from two universal premises. Thus we have a list of eight strengthened syllogisms, *viz.* AAI and EAO in fig. 1; EAO, AEO in fig. 2; AAI, EAO in fig. 3; AAI and EAO in fig. 4. We know that four of these are subaltern moods or weakened syllogisms, *viz.* AAI, EAO in fig. 1; EAO, AEO in fig. 2. In addition to these four, there is another weakened syllogism, *viz.* AEO in the fourth figure. Since in each of the subaltern moods we can instead of a particular conclusion put its subaltern universal as the conclusion, these moods are supposed to be worthless and misleading. But if we are to mention the possible valid moods of all the figures, we should not omit subaltern moods, and should name all the 24 possible valid moods. We have already pointed out that if we omit the weakened syllogisms of all the figures, we are left with nineteen valid moods in all. If we omit all the strengthened and weakened syllogisms of all the figures, we are left with only fifteen valid moods.

Aristotle, as we have already remarked, regarded the first figure as the perfect figure. It is undeniable that the first figure is the most natural, and by far the most important of all the figures. In it, the subject of the conclusion is also the subject of the minor premise, and the predicate of the conclusion is the predicate of the major premise. Thus the meaning of S and P, the minor and the major terms, is the same in the premises and in the conclusion. If S is read in denotation in the minor premise, it may be read in denotation in the conclusion; and if P is read in connotation in the major premise, it can be read in connotation in the conclusion. Further, it is natural that the predicate of a proposition should have greater extension than the subject. The 'heavier' term, says Bosanquet, is naturally the predicate of a proposition. 'All men are mortal' is a more natural proposition than 'Some mortals are men.' In the first figure this condition is fulfilled, and the predicate in each of the propositions of a syllogism in this figure has usually greater extension than the subject. Furthermore, in this figure alone the major premise states the principle and the minor premise gives an instance of it. This is the only figure which can prove A, E, I, O, that is, all kinds of propositions. Besides, from the scientific point of view an A proposition is the most important, since by means of it we can state a law, and the first figure is the only figure which can prove an A proposition.

The first figure the most natural.

Though the first figure is the most important, some arguments naturally fall in other figures. The second

The uses of each of the four figures of the syllogism.

figure proves only negative conclusions, and is called the exclusive figure. While the **first** figure is useful for **discovery** and **proof**, the **second** is useful for the purposes of **disproof**, because **it proves negatives.** By means of it we may go on excluding various suppositions as to the nature of the object under investigation, whose real character we wish to ascertain. Such an argument as, Such and such orders have such and such properties, This plant has not those properties, ∴ It does not belong to that order, falls naturally in the second figure. The **third** figure **proves** only **particulars.** By means of it we can **disprove universal** propositions. By proving O we can disprove A, and by proving I we can disprove E. It is a natural figure when the subject of the premises is a singular term, *e.g.* Socrates is a Greek, Socrates is a philosopher, ∴ Some philosophers are Greeks. The fourth figure is supposed to be unnatural, but arguments can be proved in this figure as in any other figure. The **fourth** figure is said to be useful for **discovery** or **exclusion** of the different **species of a genus.** Lambert sums up the utility of the different figures in the following words:—"The first figure is suited to the discovery or proof of the properties of a thing; the second to the discovery or proof of the distinctions between things; the third to the discovery or proof of instances and exceptions; the fourth to the discovery or exclusion of the different species of a genus."

The fourth figure was not recognised by Aristotle, on the ground that it was unnatural. We have already

Should the fourth figure be recognised as a distinct figure?

remarked that this figure can prove its conclusions as faultlessly as any other figure. However it is very seldom that arguments fall naturally in this figure. This figure is the reverse of the first figure. If we transpose the premises of the first figure, we get the form of the fourth figure as determined by the position of the middle term in relation to the extremes. The form of the premises of the first figure is M—P, S—M; transposing these, we get S—M, M—P, which is of the same form as the fourth figure. If this is so, then if from a mood of the first figure we draw P—S as the conclusion instead of S—P, by transposing its premises, then we have a mood in the fourth figure. Aristotle held that some moods in the first figure allow us to draw P—S as the conclusion. If we do so, the conclusion is indirect, and such a mood he calls an indirect mood. Let us now see what moods may yield an indirect conclusion in the first figure.

From All men are mortal, All philosophers are men, we may legitimately draw the indirect conclusion, Some mortals are philosophers. If we do so, the mood becomes Bramantip in the fourth figure. Similarly, from No men are perfect, All philosophers are men, we may draw the indirect conclusion, No perfect beings are philosophers. Here we have the mood Camenes in the fourth figure. Again, from the premises All roses are sweet-scented, Some flowers are roses, we may draw the indirect conclusion, Some sweet-scented things are flowers. In this case we have the mood Dimaris in the fourth figure. From the premises All virtuous men are trusted, No politicians are

virtuous, we may draw the indirect conclusion, Some that are trusted are not politicians. In this case we have the mood Fesapo in the fourth figure. Lastly, from the premises Some flowers are white, No oranges are flowers, we may draw the indirect conclusion, Some white things are not oranges. This is the mood Fresison in the fourth figure. Thus by drawing indirect conclusions from valid and invalid moods of the first figure, Aristotle provided all the moods of the fourth figure, *viz.* Bramantip, Camenes, Dimaris, Fesapo, Fresison. But if we recognise indirect moods of the first figure, there is no harm in recognising a fourth figure which can directly yield the same conclusions.

Johnson points out that the fourth figure cannot be proved to be unnatural. Some arguments do appear in this figure. He points out that the worthlessness of the fourth figure is ordinarily proved by the argument—"Any argument worthy of logical recognition must be such as would occur in ordinary discourse. Now it will be found that no argument occurring in ordinary discourse is in the fourth figure. Hence, no argument in the fourth figure is worthy of logical recognition". But this very argument, he points out, is in the fourth figure. So it cannot be established that the fourth figure is useless. The following example given by Keynes falls naturally in the fourth figure: None of the Apostles were Greeks, Some Greeks are worthy of all honour, ∴ Some worthy of all honour are not Apostles. If it be said that the fourth figure is nothing but the first with a converted conclusion, the remark does not apply to Fesapo and Fresison, for there is no valid mood of the first figure the conversion of whose conclu-

sion can provide the conclusions required by them. Besides, if we define figure as the form of the syllogism as determined by the position of the middle term, then the place of the fourth figure cannot be denied.

Pure Hypothetical and Disjunctive Syllogisms

Having ascertained the nature of pure categorical syllogisms, we may now discuss briefly pure **hypothetical** and **disjunctive syllogisms.** The same principles which apply to pure categorical syllogisms, apply to these syllogisms as well, and no new principles are involved. A distinction is often drawn between **hypothetical** and **conditional** propositions. The form of a hypothetical proposition is If P then Q, while that of a conditional proposition is If S is M, it is P. In a **conditional** proposition there is a **common element between** the **antecedent** and the **consequent,** while there is **no** such **common element** between them in a **hypothetical** proposition. But since the same principles are involved in both hypothetical and conditional syllogisms, we need not treat of them separately. The **hypothetical** proposition **is abstract** in form, while the **conditional** proposition **may be concrete.** Further we have previously remarked that the traditional fourfold distinction of propositions applies clearly to hypothetical propositions in their conditional or denotative form, which is concrete.

Ignoring the distinction between hypothetical and conditional propositions, we may define a pure **hypothetical syllogism** as a reasoning consisting of **two hypo-**

Hypothetical syllogisms explained and illustrated.

thetical premises and a **hypothetical conclusion.** We may now illustrate some valid moods by means of hypothetical syllogisms. Barbara—If any S is M, always that S is P, If any S is Q, always that S is M, therefore If any S is Q, always that S is P. The following is a concrete example : If any person is cruel, always he is hated ; If any person recklessly wounds the feelings of another, always he is cruel ; therefore If any person recklessly wounds the feelings of another, always he is hated. Cesare—If any S is M, never that S is P, If any S is Q, then always that S is P, therefore If any S is Q, never that S is M. If any man is foolish, he is never respected ; If any man is selfless, he is always respected ; therefore If any man is selfless he is never foolish. Bocardo—If an S is M, then sometimes not that S is P, If any S is M, then always that S is Q, therefore If an S is Q, then sometimes not that S is P. If a state is weak, sometimes it is not feared ; If any state is weak, always it fails to preserve its independence ; therefore If a state fails to preserve its independence, sometimes it is not feared. Bramantip—If any S is M, always it is P, If any S is P, always it is Q, therefore If an S is Q, sometimes it is M. If any currency is inflated, always it uses inconvertible paper money ; If any currency uses inconvertible paper money, always it loses respect in the foreign market ; therefore If a currency loses respect in the foreign market, it is sometimes inflated. We need not multiply more examples of hypothetical syllogisms ; the above are sufficient to exhibit their nature.

We know that the force of any syllogistic argument

depends upon the necessity with which the conclusion follows from the premises. This necessity can be expressed by both hypothetical and categorical syllogisms. So conditional propositions can be reduced to categorical ones. Thus the argument, If any man is cruel, he is always hated ; If any man is cruel, he is always selfish ; therefore If a man is selfish he is sometimes cruel, can be reduced to the categorical syllogism, A cruel man is hated, A cruel man is selfish, therefore A selfish man is hated. We must remember that Barbara is the only form of the hypothetical syllogism which is of much importance. since hypothetical propositions are by nature universal and abstract.

The possibility of the disjunctive syllogism considered.

Logicians have not generally considered the possibility of disjunctive syllogisms, but in some cases such syllogisms are undoubtedly possible. Since they can never be negative, the only disjunctive syllogisms possible are those in which the conclusion and premises are affirmative propositions. Barbara is the only form of the disjunctive syllogism which is of any importance. In a disjunctive syllogism we can secure a middle term only when the minor premise negatives one of the alternatives of the major. Thus the following disjunctive syllogism is in the mood Barbara :—S is either P or Q, S is either P′ or R, ∴ S is either Q or R. The argument will be apparent if we reduce it to the hypothetical form : If S is P′ it is Q, If S is R′ it is P′, ∴ If S is R′, it is Q. The following argument is *not* a disjunctive syllogism : S is either P or Q, S is either P or R, ∴ S is either P or Q or R.

Reduction of Syllogisms

"By **reduction** is meant a process whereby the reasoning contained in a given syllogism is expressed in some other mood or figure. Unless an explicit statement is made to the contrary, reduction is supposed to be to figure 1" (Keynes).

Reduction regarded as indispensable by Aristotle.

Aristotle, we have already stated, believed that the dictum de omni et nullo is the ground of all syllogistic inference. But this dictum, which asserts that whatever can be affirmed or denied of a class distributively can be, in the same way, affirmed or denied of anything belonging to that class, applies directly to the first figure only, the empty scheme of which is M—P, S—M, S—P. We have seen that the dictum requires that the major premise should be universal and the minor premise affirmative, and that the conclusion should be affirmative if the major premise is affirmative and negative if it is negative. We have also pointed out that the general rules of the syllogism can be deduced from the dictum by generalising it. Aristotle and his followers however thought that since the dictum applies directly to the first figure only, this is the perfect figure, and the moods of other figures should be reduced to the moods of the first figure to establish their validity. So the doctrine of reduction has become a part of the syllogistic doctrine.

Reduction may be either **direct** or **indirect.** In the case of **direct** reduction a **mood** is **changed** into some

Direct and indirect reduction explained.

other mood with the help of the processes of **immediate inference.** Thus Cesare can be converted to Celarent by converting the major premise. The syllogism No P is M, All S is M, ∴ No S is P is in the mood Cesare; by converting its major premise we get the syllogism No M is P, All S is M, ∴ No S is P, which is in the mood Celarent. Aristotle reduced Baroco and Bocardo indirectly, because these moods cannot be reduced to the moods of the first figure without the introduction of negative terms, that is, without taking the help of obversion. But Aristotle was averse to the introduction of negative terms. So he reduced them **indirectly.** The indirect method of proof was frequently adopted by Euclid in his Elements of Geometry. Indirect reduction is an effective weapon in controversy, and is perhaps one of the methods most commonly employed for that purpose.

Indirect reduction of Baroco.

Let us see how Baroco and Bocardo can be reduced indirectly to the first figure. Take Baroco, which is All P is M, Some S is not M, ∴ Some S is not P. Suppose this argument is not valid. Now, the truth of the premises of the argument cannot be questioned, since it is a syllogism, the premises of which are always accepted as true. Thus the falsity of the argument must be due to the falsity of the conclusion. If the conclusion Some S is not P is not true, then according to the law of excluded middle its contradictory, All S is P, must be true. Now, if this is so, then the propositions All P is M, Some S is

not M, All S is P are true together. Now combining All S is P with the original major premise, we get the syllogism, All P is M, All S is P, ∴ All S is M. This argument is in the mood Barbara, and therefore its validity cannot be questioned. But if the conclusion established here, *viz.* All S is M, is true, then its contradictory, Some S is not M, must be false, according to the principle of contradiction. But Some S is not M cannot be false, because it is the minor premise of the original syllogism and has been given as true. Therefore the new minor premise, All S is P, must be false, and its contradictory, Some S is not P, true. Therefore the original argument, All P is M, Some S is not M, ∴ Some S is not P, is a valid argument. Such an indirect reduction is called **Reductio ad impossibile,** or Reductio per impossibile, **Deductio** ad impossibile or Deductio **ad absurdum.**

Indirect reduction of Bocardo.

We may now reduce Bocardo indirectly to figure 1. The argument is, Some M is not P, All M is S, ∴ Some S is not P. If this argument is not true, then the conclusion must be false, since the truth of the premises cannot be questioned. If the given conclusion is false, then its contradictory, All S is P, must be true, according to the principle of excluded middle. Then the propositions Some M is not P, All M is S, All S is P are true together. Combining All S is P with the original minor premise we get the syllogism, All S is P, All M is S, ∴ All M is P. Here the process of reasoning is perfect, the argument being in Barbara. But if the new conclusion is true, then its contradictory, Some M is not P, must be

false, according to the law of contradiction. But Some M is not P cannot be false, since it is the major premise of the original syllogism. Therefore the new major premise, All S is P, must be false, and its contradictory, Some S is not P, the conclusion of the original syllogism, true. Therefore the original syllogism, Some M is not P, All M is S, ∴ Some S is not P, is valid.

We have already pointed out that certain mnemonics have been adopted by logicians to name the valid moods of different figures. They are—

Barbara, Celarent, Darii, Ferio (Fig. 1);

Cesare, Camestres, Festino, Baroco (Fig. 2);

Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison (Fig. 3);

Bramantip, Camenes, Dimaris, Fesapo, Fresison (Fig. 4).

This list omits the five subaltern moods, and names the nineteen fundamental valid moods of different figures. De Morgan calls the mnemonic lines magic words, full of meaning, and by these words valid moods of different figures have been denoted for many centuries. The only meaningless letters of the mnemonic words are *b* (not initial), *d* (not initial), *l*, *n*, *r*, *t*. The vowels of each word give the quality and quantity of the propositions of which the syllogism is composed. The initial letters in the case of figures 2, 3 and 4 show to which of the moods of figure 1 the given mood is to be reduced. It is to be reduced to that mood of the first figure which has the same initial letter. Thus Camestres is reduced to Celarent,

The meaning of the letters of the mnemonic words.

Darapti to Darii, Fesapo to Ferio, Bramantip to Barbara and so on. *s* (in the middle of a word) indicates that in the process of reduction the preceding proposition is to be simply converted. *s* (at the end of a word) shows that the conclusion of the new syllogism has to be simply converted in order that the given conclusion may be obtained. *p* (in the middle of a word) signifies that the preceding proposition is to be converted per accidens, *e.g.*, in the reduction of Darapti to Darii. *p* (at the end of a word) implies that the conclusion obtained by reduction is to be converted per accidens to get back to the original conclusion. *m* indicates that in reduction the premises have to be transposed (metathesis præmissarum), as in the case of Camestres or Bramantip. *c* signifies that the mood is to be reduced indirectly (*i.e.*, by reductio per impossibile, as shown before). The position of the letter *c* indicates that we are to omit the premise preceding it, that is, the other premise is to be combined with the contradictory of the conclusion.

Direct reduction of Baroco and Bocardo.

We have already considered how Baroco and Bocardo were reduced indirectly, but logicians have reduced them directly as well. In the case of direct reduction, Faksoko is sometimes given as a mnemonic for Baroco, and Doksamosk for Bocardo. Here *k* denotes obversion, *ks* obversion followed by conversion, that is, contraposition, and *sk* conversion followed by obversion. Let us now reduce Baroco and Bocardo directly. The mood Baroco (Faksoko) may be represented by the syllogism All P is M, Some S is not M, therefore Some S is not P. Now contraposing the major premise of the syllogism

and obverting the minor premise, we get the following syllogism: No not-M is P, Some S is not-M, therefore Some S is not P, which is in the perfect mood Ferio. Similarly the mood Bocardo (Doksamosk)—Some M is not P, All M is S, ∴ Some S is not P—can be reduced to Darii by contraposing the major premise of the syllogism and transposing the premises. The syllogism will then stand as All M is S, Some not-P is M, ∴ Some not-P is S. Now by first converting this conclusion and then obverting, as required by *sk* at the end of Doksamosk, we get the original conclusion, Some S is not P.

Direct reduction of 2nd, 3rd and 4th figures.

We may now **directly reduce** some of the moods of figures 2, 3 and 4 to the moods of the first figure. Camestres—All P is M, No S is M, ∴ No S is P—can be reduced to Celarent by simply converting the minor premise and transposing the premises. When thus reduced the syllogism will become No M is S, All P is M, ∴ No P is S. Now simply converting the new conclusion, as required by *s* at the end of Camestres, we get the original conclusion, No S is P. Darapti—All M is P, All M is S, ∴ Some S is P—can be reduced to Darii by converting the minor premise by limitation. The syllogism will then become All M is P, Some S is M, ∴ Some S is P. Disamis—Some M is P, All M is S, ∴ Some S is P—may be reduced to the syllogism All M is S, Some P is M, ∴ Some P is S, which is in the mood Darii, by simply converting the major premise of Disamis and transposing the premises. The new conclusion, Some P is S, can be simply converted to Some S is P, as required by *s* at the end of Disamis. Bramantip—All P is M, All

M is S, ∴ Some S is P—can be reduced to Barbara by transposing the premises. The syllogism will then become All M is S, All P is M, ∴ All P is S. By converting this new conclusion per accidens, as required by *p* at the end of Bramantip, we get the original conclusion, Some S is P. Fesapo—No P is M, All M is S, ∴ Some S is not P—can be reduced to Ferio by converting the minor premise per accidens and the major premise simply. The syllogism will then become No M is P, Some S is M, ∴ Some S is not P. We need not reduce other moods directly to the moods of the first figure, the above examples being sufficient to show the principles involved.

Indirect reduction of Darapti.

Though Aristotle reduced only Baroco and Bocardo indirectly, other moods of figures 2, 3 and 4 may also be reduced **indirectly** to the moods of the first figure. We may here give only one example by way of illustration. Let us take Darapti—All M is P, All M is S, ∴ Some S is P. If the conclusion, Some S is P, is not true, then its contradictory, No S is P, must be true. Then the propositions All M is P, All M is S, and No S is P will be true together. Now combining No S is P with the original minor premise we get the syllogism No S is P, All M is S, ∴ No M is P. This argument is in the perfect mood Celarent. If the new conclusion is valid, then its contrary, All M is P, becomes invalid, because we know that of two contrary premises if one is true, the other is false according to the principle of contradiction. But All M is P, which is the major premise of the original syllogism, cannot be false. There is no fallacy in the new argument, it being in the perfect mood Celarent. The new minor premise,

which is the original minor premise, cannot be false. Therefore the new major premise, No S is P, which is the contradictory of the conclusion of the original syllogism, must be false, and therefore the original conclusion, Some S is P, must be true. Therefore the original syllogism, All M is P, All M is S, ∴ Some S is P, is true.

A mood of a figure may be reduced to another mood of that figure.

We may also point out that a mood of a figure may be reduced to another mood of the same figure. Thus Barbara, All M is P, All S is M, ∴ All S is P, may be reduced to Celarent by the obversion of the major premise and of the conclusion. The syllogism will then become No M is not-P, All S is M, ∴ No S is not-P. Similarly Celarent may be reduced to Barbara by obverting the major premise and the conclusion. Darii may be reduced to Ferio and Ferio to Darii by obverting the major premise and the conclusion. Barbara and Darii are reducible to each other by means of the indirect method. We need not however show how this is possible.

Should the problem of reduction be treated of in connection with the doctrine of the syllogism?

Whately, following Aristotle, argues that since the figures 2, 3 and 4 are imperfect, the moods of these figures can be proved valid by reducing them to the moods of the first figure. So according to him we cannot but consider reduction in the treatment of the syllogism. Fowler holds that though the validity of the moods of figures 2, 3, and 4 can be proved by other dicta which are as self-evident as the dictum of Aristotle, yet if we reduce them to moods of the first figure we can be sure of their validity. Ueberweg

appears to be right when he points out that to prove the validity of the moods of syllogisms we do not require the help of any axiom whatsoever. The validity of particular moods is as self-evident as the validity of the axioms themselves. It is undeniable that the axioms are deductions from the nature of syllogistic reasoning itself, and its validity does not depend upon them. It is also true that if we depend upon axioms to prove the validity of particular pieces of syllogistic reasoning, we need not depend upon the axiom of Aristotle alone. Every figure has its independent self-evident axiom, as we have already shown. We have also seen that some arguments fall naturally in figures 2, 3 and 4 instead of figure 1. In spite of all this, it appears that the treatment of reduction as a part of the doctrine of the syllogism is useful, inasmuch as it shows the equivalence between the moods in different figures, and thus reminds us that syllogistic reasoning is a whole, a unity, though it admits of being separated into parts, or plurality. Further, reduction is an excellent logical exercise.

Reduction of hypothetical syllogism.

A hypothetical syllogism can be reduced in the same way as a categorical syllogism. It will be best to take an example in the concrete form. Cesare—If any S is M, then never that S is P; If any S is Q, then always that S is P; ∴ If any S is Q, then never that S is M—can be reduced to Celarent by converting the major premise of the syllogism; thus the new syllogism will be, If any S is P, then never that S is M; If any S is Q, then always that S is P; ∴ If any S is Q, then never that S is M. Darapti—If any S is P, then always that S is M;

If any S is P, then always that S is Q ; ∴ If an S is Q, then sometimes that S is M—can be reduced to Darii by converting its minor premise per accidens ; the syllogism when reduced becomes, If any S is P, that S is M ; If an S is Q, then sometimes that S is P ; ∴ If an S is Q, then sometimes that S is M. Bramantip—If any S is M, then always that S is P ; If any S is P, then always that S is Q ; ∴ If an S is Q, then sometimes that S is M—can be reduced to Barbara by transposing the premises ; the new syllogism will be, If any S is P, then always that S is Q ; If any S is M, then always that S is P ; ∴ If any S is M, then always that S is Q. The new conclusion, If any S is M, then always that S is Q, has to be converted per accidens, as required by *p* at the end of Bramantip, to get back to the original conclusion, which is, If an S is Q, then sometimes that S is M. Bocardo (Doksamosk)—If an S is P, then sometimes not that S is M; If any S is P, then always that S is Q ; ∴ If an S is Q, then sometimes not that S is M—can be reduced to Darii by contraposing the major premise and transposing the premises ; the new syllogism will then be, If any S is P, then always that S is Q ; If an S is not-M, then sometimes that S is P; ∴ If an S is not-M, then sometimes that S is Q. If the new conclusion is first converted and then obverted, as required by *sk* at the end of Doksamosk, we get back to the original conclusion, *viz.* If an S is Q, then sometimes not that S is M. We need not consider the indirect reduction of hypothetical syllogisms, though they may be indirectly reduced in the same way as categorical syllogisms.

CHAPTER III

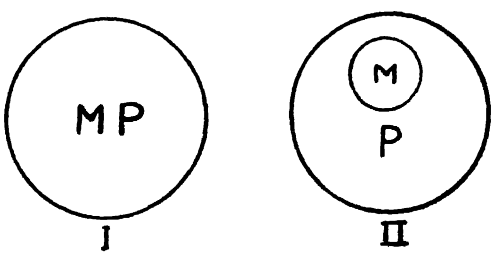
DIAGRAMMATIC REPRESENTATION OF SYLLOGISMS

Euler's Diagrams may be used to illustrate syllogistic reasonings. The method of procedure will be understood if we first apply them to a syllogism in Barbara,—

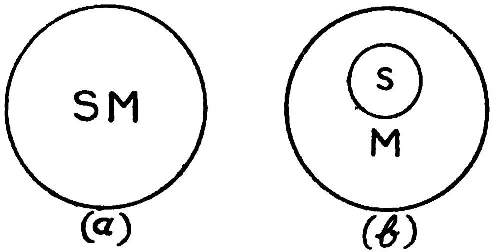
All M is P,
All S is M,
All S is P.

First the premises must be represented separately by means of the appropriate diagrams. In this syllogism each premise is an A proposition, which requires two diagrams to represent it. Thus we have—

M a P,

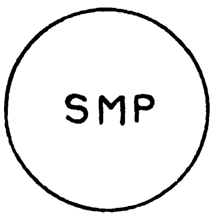


S a M,

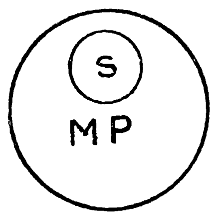


To obtain the conclusion, each of the cases yielded by the major premise must be combined with each of those yielded by the minor. Thus we have following diagrams—

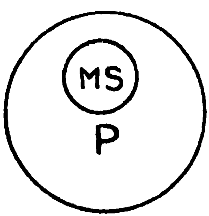
I and (a) give—



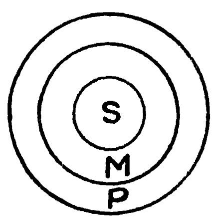
I and (b) give—



II. and (a) give—



II. and (b) give—



To find the conclusion, we have to ask what relation between S and P is common to all these four diagrams? The answer is plainly, that in the first of them (I (a)) S and P are co-extensive, and in the other three the

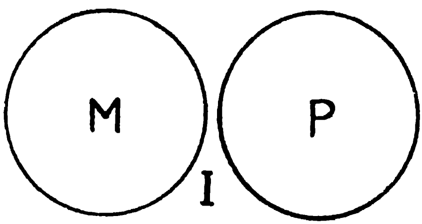
class S is entirely included within the class P. In all four cases, therefore, All S is P. We proceed to illustrate the other three valid moods of the first figure.

Celarent.—

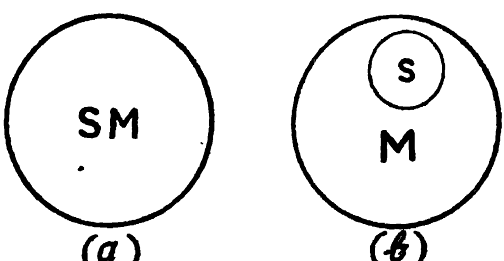
No M is P,
All S is M,
No S is P.

Here the major premise requires only one diagram, the minor two. Thus,—

M e P,



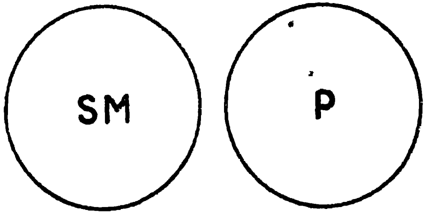
S a M



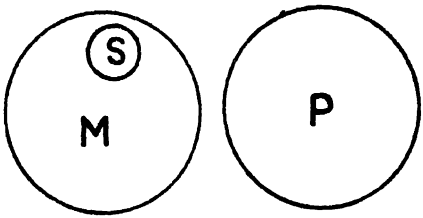
(a) (b)

In combination,

I. and (a) give—



I. and (b) give—



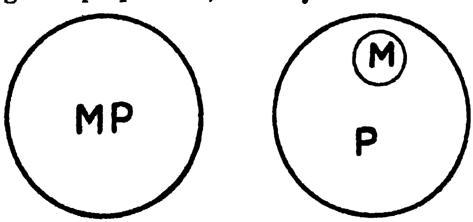
Asking now what is the relation between S and P, we see that in both cases the whole of S is excluded from the whole of P ; *i.e.* No S is P.

Darii.—

All M is P,
Some S is M,
∴ Some S is P.

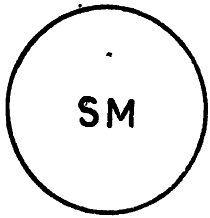
Here the major premise requires two diagrams, the minor, being an I proposition, as many as four. Thus,—

M a P,

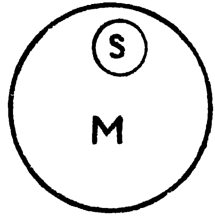


I II

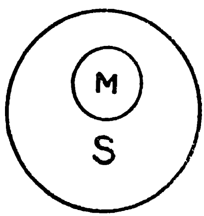
S i M,



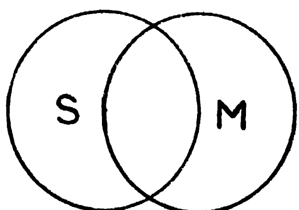
(a)



(b)



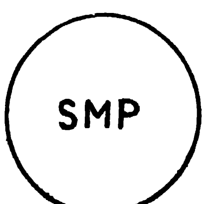
(c)



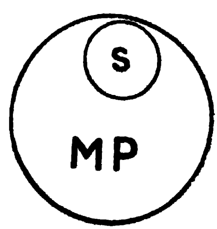
(d)

When we combine these, we find that II. and (c) themselves give four alternative diagrams, and II. and (d) two alternatives. The total combinations are as follows :—

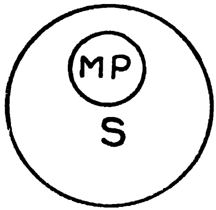
I. and (a) give—



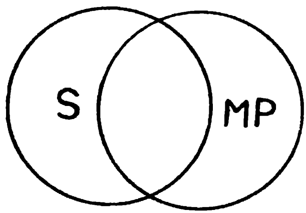
I. and (b) give—



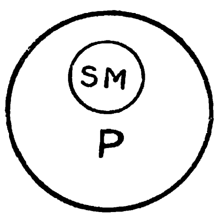
I. and (c)
give—



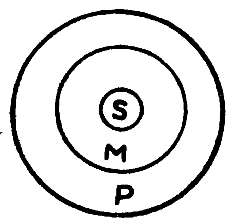
I. and (d)
give—



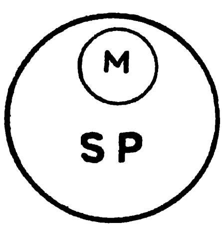
II and (a)
give—



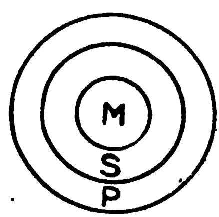
II. and (b)
give—



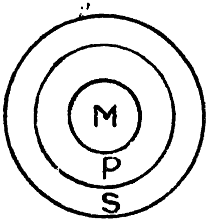
II. and (c)
give—



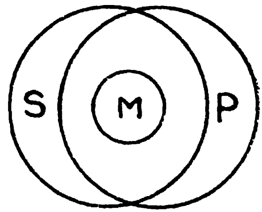
or



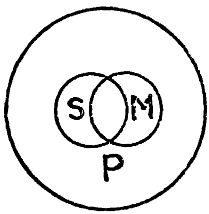
or



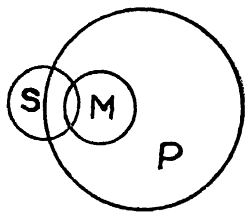
or



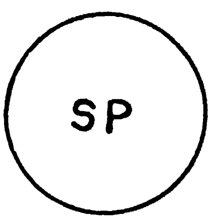
II. and (d) give—

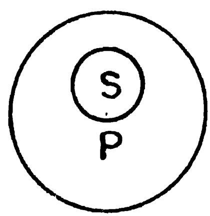


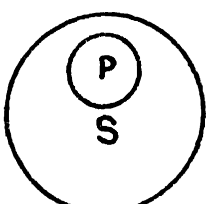
or

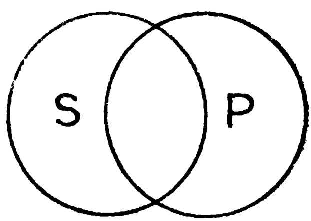


To find the conclusion, if we leave M out of account, we find that the above diagrams reduce to the following four, *viz.*—









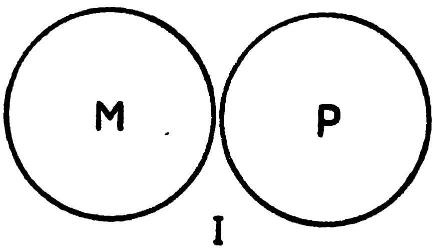
and what is common to *all* these four cases is that some members of the class S coincide with some members of the class P. That is, Some S is P.

Ferio.—

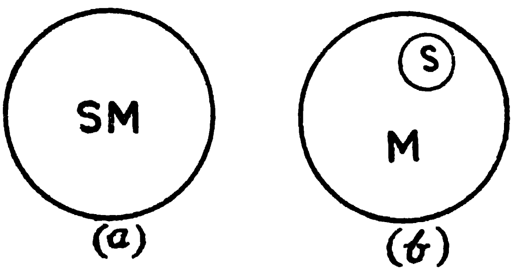
No M is P,
Some S is M,
∴ Some S is not P.

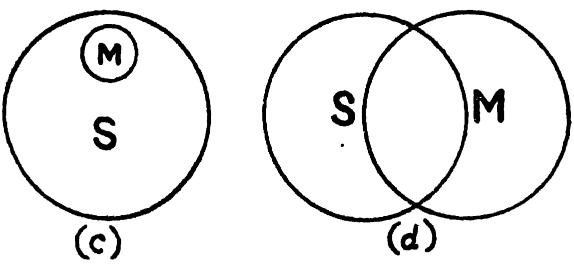
Here we have—

M e P,



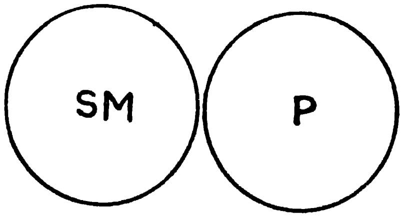
S i M,



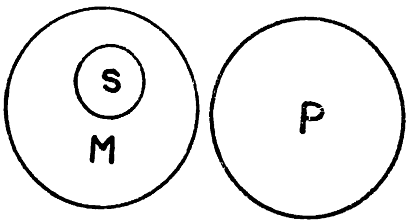


This gives the following combinations :—

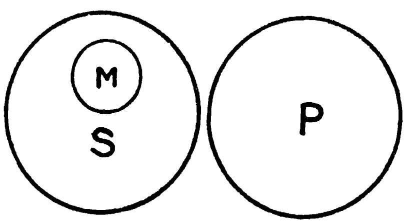
I. and (a)
give—



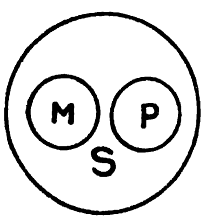
I. and (b)
give—



I. and (c)
give—



or



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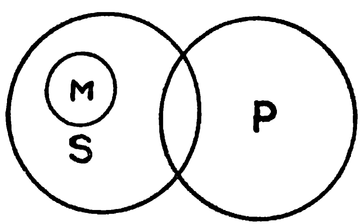
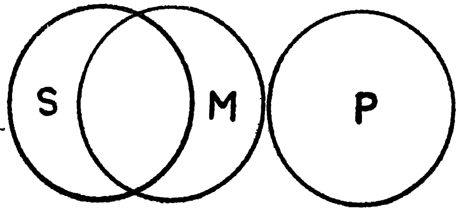
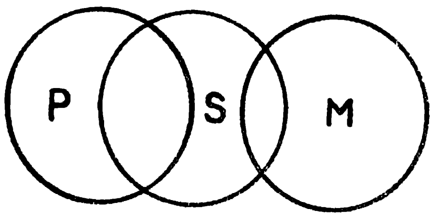


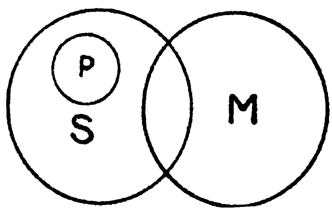
I. and (d) give—



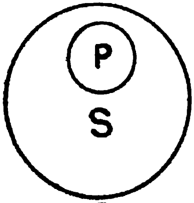
or



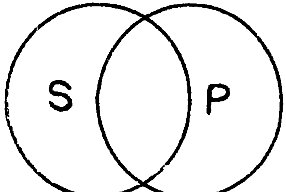
or



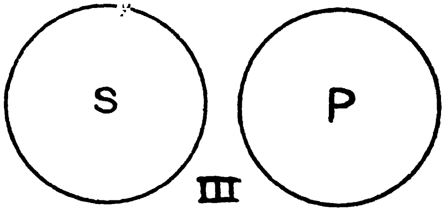
If we now leave M out of account, we find that the above diagrams reduce to the following three, *viz.*—



I



II



III

and what is common to *all* these three cases is that some members of the class S are excluded from the class P; *i.e.* Some S is not P.

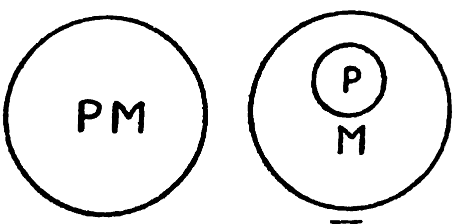
We also give an example of one mood in each of the remaining figures.

In the second figure take *Baroco*,—

All P is M,
Some S is not M,
∴ Some S is not P.

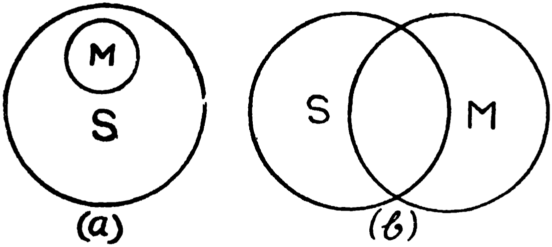
Here we have—

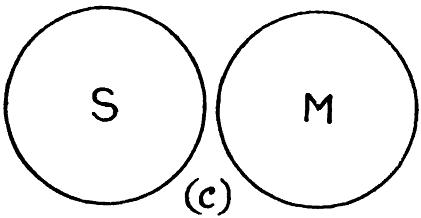
P a M,



I II

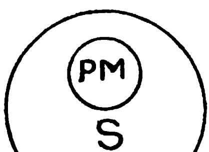
S o M,



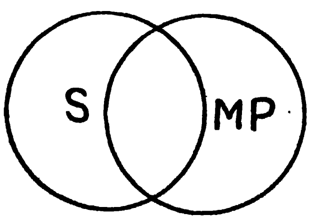


Then,

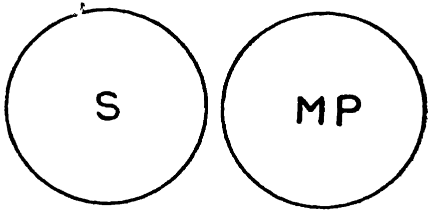
I. and (a) give—



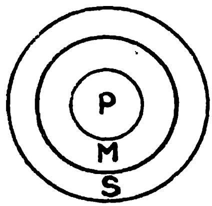
I. and (b) give—



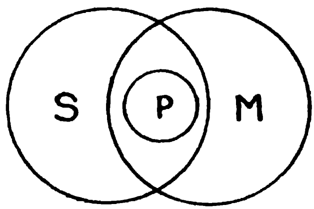
I. and (c)
give—



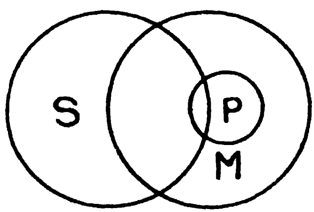
II. and (a)
give—



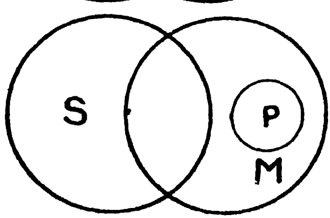
II. and (b)
give—



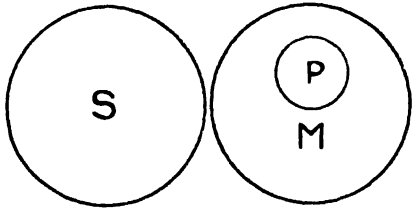
or



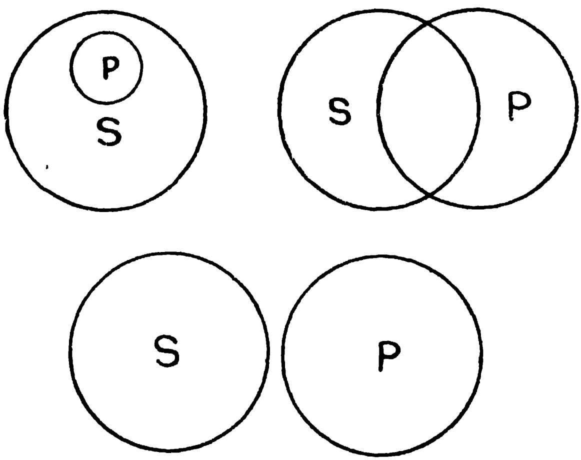
or



II. and (c) give—



If we now leave M out of account, we find that the relations between S and P in all these diagrams are expressed in the three figures—



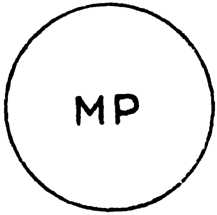
That is, the conclusion is, Some S is not P.

In the third figure, take *Disamis*—

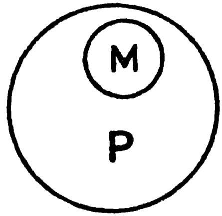
Some M is P,
All M is S,
Some S is P.

Here we have,

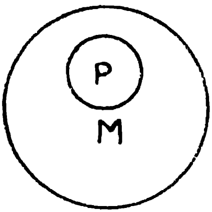
M i P,



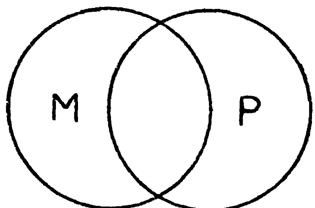
I



II

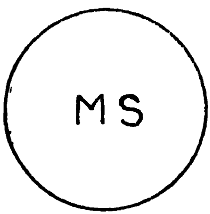


III

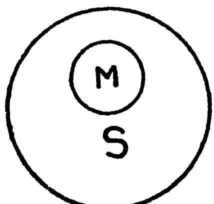


IV

M a S,



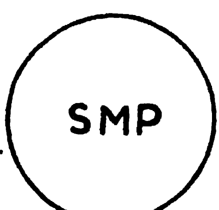
(a)



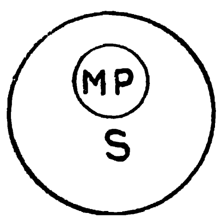
(b)

Then,

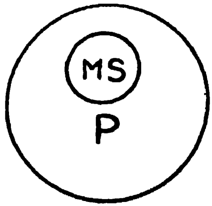
I. and (a) give—



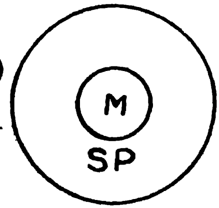
I. and (b) give—



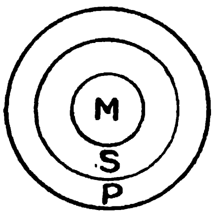
II. and (a) give—



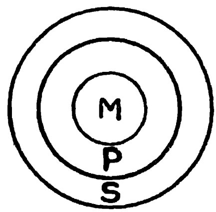
II. and (b) give—



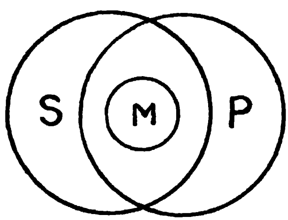
or



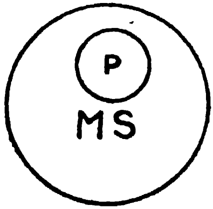
or



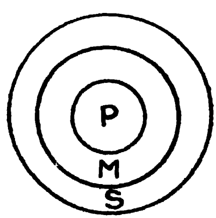
or



III. and (a) give—

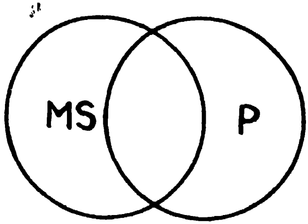


III. and (b) give—



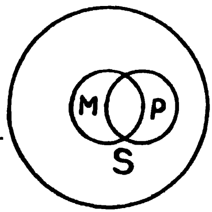
IV. and

(a) give—

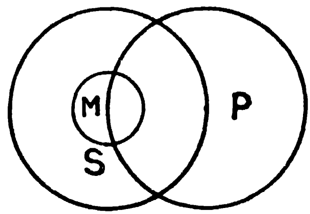


IV. and

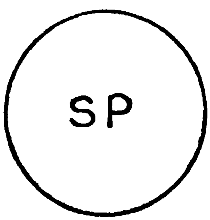
(b) give—

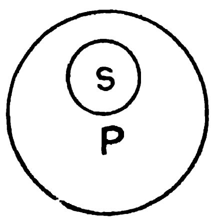


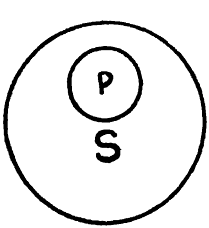
or

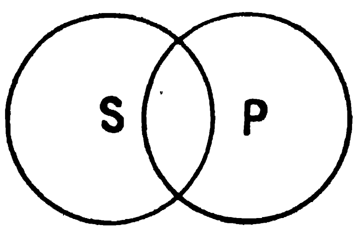


If we neglect M, the above diagrams reduce to four only, *viz.*—









and what is common to all four is that some members of

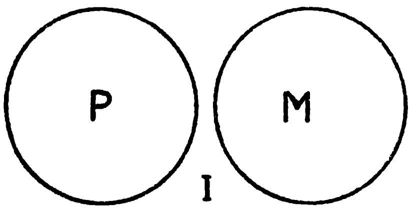
the class S are included in the class P. That is, Some S is P.

In the fourth figure, take *Fresison*,

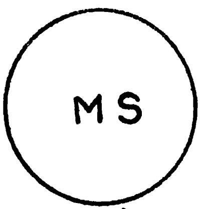
No P is M,
Some M is S,
∴ Some S is not P.

Here we have,

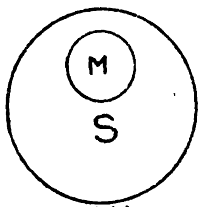
P e M,



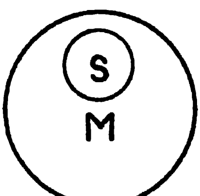
M i S,



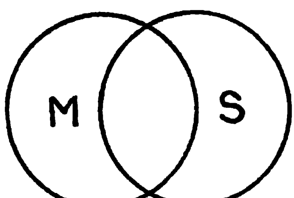
(a)



(b)



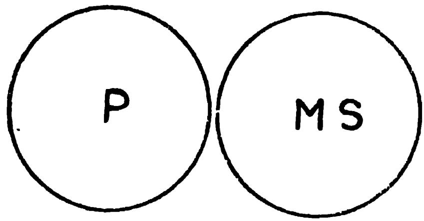
(c)



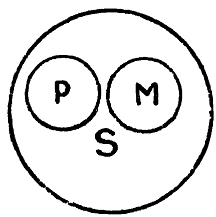
(d)

Then,

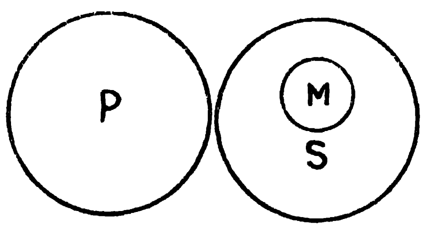
I. and (a) give—



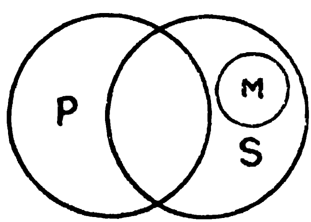
I. and (b) give—



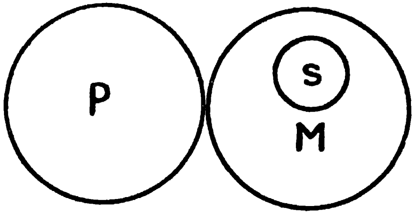
or



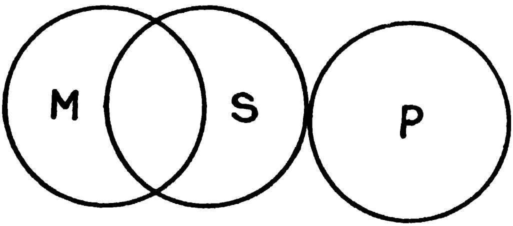
or



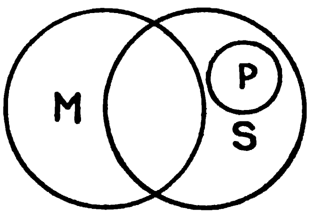
I. and (c) give—



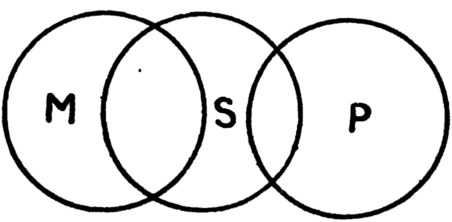
I. and (d) give—



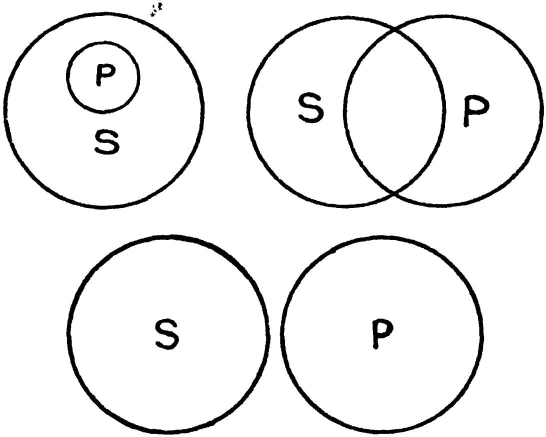
or



or



Neglecting M, we find that the above diagrams (which will be found to be the same as the diagrams of *Ferio*, though in a different order) reduce to three, *viz.*—



from which, as in *Ferio*, we are justified in concluding that Some S is not P.

CHAPTER IV

MIXED SYLLOGISMS

Mixed Hypothetical Syllogisms

We may now discuss the nature of **mixed syllogisms.** We have already noted that we shall discuss mixed syllogisms under three main heads, *viz.* (1) **Mixed Hypothetical** syllogisms, (2) **Mixed Disjunctive** syllogisms, and (3) the **Dilemma.** Jevons is of opinion that mixed hypothetical syllogisms should be called simply hypothetical syllogisms, since categorical and pure hypothetical syllogisms need not be distinguished, as they are governed by the same principles. Some call mixed hypothetical syllogisms hypothetico-categorical syllogisms. Fowler, Sigwart, Keynes, Welton and others distinguish between pure hypothetical and mixed hypothetical syllogisms. The former involve the same principles as categorical syllogisms, and we have seen that in them all the propositions are hypothetical, whereas mixed hypothetical syllogisms are governed by new principles. We therefore adopt this distinction between pure and mixed syllogisms, to avoid confusion.

Preliminary remarks.

A **mixed hypothetical syllogism** is one in which the **major premise** is a **hypothetical** proposition and the **minor premise** and the **conclusion** are **categorical propositions.** To illustrate a mixed hypothetical syllogism we may express the major

The nature of the mixed hypothetical syllogism explained and illustrated.

premise either by a hypothetical proposition or by a conditional one. The following is an example in which the major premise is an abstract hypothetical proposition: If P is true, Q is true (major premise); P is true (minor premise); ∴ Q is true (conclusion). In the following example the major premise is a conditional proposition: If any S is M, it is P (major premise); This S is M (minor premise); ∴ This S is P (conclusion). The following is a concrete example: If any man takes an overdose of strychnine he dies; Philip has taken an overdose of strychnine; ∴ He will die. In each of the above examples the minor premise affirms the antecedent, and the conclusion affirms the consequent. There is another form of mixed hypothetical syllogism in which the minor premise denies the consequent, and the conclusion denies the antecedent. Of this form the following are examples: If P is true, Q is true; Q is not true; therefore P is not true. If any S is M, it is P (major premise); This S is not P (minor premise); therefore This S is not M (conclusion). If any man is honest he is trusted; Ranjit is not trusted; therefore Ranjit is not honest.

The two canons of mixed hypothetical syllogisms are:

Canons of the mixed hypothetical syllogism and fallacies arising from their violation.

(1) The **assertion** of the **truth** of the **antecedent** of a hypothetical proposition **justifies** the **assertion** of the **truth** of the **consequent**, but **not conversely**; (2) the **denial** of the **consequent** necessitates the **denial** of the **antecedent**, but not conversely. In every hypothetical syllogism the implication between one proposition and another is given in the major premise, and the relation between the antecedent and the conse-

quent is one of ground and consequence. The first of the above rules may be proved thus :—A particular consequent may follow from different antecedents. Thus the occurrence of Q may follow the occurrence of either A, B, or C. A man's death may be due to his taking poison or to his being attacked with some fatal illness. Now if the proposition is that If a man takes poison, he dies, we may by affirming its antecedent, *viz.* that this man has taken poison, affirm the consequent, *viz.* that this man will die ; but by affirming the consequent, *viz.* that this man will die, we cannot affirm the antecedent, that this man has taken poison, because his death may be due to some other reason, say, to his being shot or to his being attacked with some fatal illness. The violation of the first rule, that is, if we affirm the antecedent by affirming the consequent, gives rise to the fallacy of **affirming the consequent.** Similarly the second rule may be proved. Since the consequent of a hypothetical proposition may follow from different antecedents, we may by denying the consequent deny the antecedent, but we cannot by denying the antecedent deny the consequent. Thus if the proposition is, If any man gets into water, his body becomes wet, we can by denying the consequent, *viz.* that the body of the man is wet, deny the antecedent, that he has got into water. But by denying the antecedent, *viz.* that the man has got into water, we cannot deny the consequent, that his body is wet, for his body may get wet if he is exposed to the rain or somebody pours water upon his person. The violation of this rule gives rise to the fallacy of **denying the antecedent.**

The two main moods of mixed hypothetical syllogisms

are (1) **Modus ponens**, which is **constructive**, and (2) **Modus tollens**, which is **destructive**.

The two fundamental moods of the mixed hypothetical syllogism.

A syllogism is in **modus ponens** when the **minor** premise **affirms** the **antecedent** of the **hypothetical major**, and the **conclusion affirms** its **consequent**. A syllogism is in **modus tollens** when the **minor** premise **denies** the **consequent** of the **hypothetical major**, and the **conclusion denies** its **antecedent**.

In every mixed hypothetical syllogism the major premise states the principle, and the minor premise is subsumed under it. Some logicians, such as Kant, Hamilton etc., regard the inference involved in mixed hypothetical syllogisms as immediate, but Keynes holds that the burden of proof is upon those who refuse to regard these syllogisms as syllogisms proper.

Are mixed hypothetical syllogisms syllogisms proper?

Keynes also asserts that mixed hypothetical syllogisms in modus ponens are similar to the syllogisms in the first figure, because in them we pass from ground to consequence; while according to him syllogisms in modus tollens are similar to the moods in the second figure, since in them we pass from the denial of the consequence to the denial of the ground. Joseph however appears to be right when he asserts that what are usually called mixed hypothetical syllogisms are not syllogisms proper, but should rather be called hypothetical arguments, since in such a reasoning there is no middle term, and the relation between the antecedent and the consequent of the hypothetical proposition in it is one of ground and consequence, not of subject and predicate.

German logicians provide four forms of modus ponens and four of modus tollens. Each of these two moods may have the following four forms :—(1) Modus ponendo ponens, (2) Modus ponendo tollens, (3) Modus tollendo tollens, (4) Modus tollendo ponens. These names are taken from the quality of the minor premise and the conclusion. In ponendo ponens both the minor premise and the conclusion are affirmative propositions. In ponendo tollens the minor premise is affirmative and the conclusion is negative. In tollendo tollens both the minor premise and the conclusion are negative. In tollendo ponens the minor premise is negative and the conclusion is affirmative. Let us now illustrate these forms of modus ponens and modus tollens respectively. The following are examples of the four forms of the modus ponens : (1) Ponendo ponens—If A then B ; A ; therefore B. If a flower is sweet-scented it is pleasant ; This flower is sweet-scented ; therefore This flower is pleasant. (2) Ponendo tollens—If A then not B ; A ; therefore not B. If a man is dishonest he is not trusted ; This man is dishonest ; therefore This man is not trusted. (3) Tollendo tollens—If not A then not B ; not A ; therefore not B. If the finances of a country are not sound, its industries cannot prosper ; the finances of Germany are not sound ; therefore its industries cannot prosper. (4) Tollendo ponens—If not A then B ; not A ; therefore B. If the people of a country are not faithful to the government, its modification is inevitable ; the people of the Philippines are not faithful to the government ; therefore its modification is inevitable.

The four forms of the modus ponens and of the modus tollens.

The following are the four forms of the modus tollens: (1) Ponendo ponens—If not A then not B ; B ; therefore A. If a man is not intelligent he is not respected ; Mr. Shastri is respected ; therefore He is intelligent. (2) Ponendo tollens—If A then not B ; B ; ∴ not A. If Japan disobeys the decision of the League of Nations, it will not secure the co-operation of other states ; Japan has secured the co-operation of other states ; therefore Japan has not disobeyed the decision of the League of Nations. (3) Tollendo tollens—If A then B ; not B ; therefore not A. If England maintains friendly relations with Germany, she will lose the co-operation of France ; England has not lost the co-operation of France ; therefore England has not maintained friendly relations with Germany. (4) Tollendo ponens—If not A then B ; not B ; therefore A. If the market does not maintain a steady price, the traders are in difficulty ; The traders are not in difficulty ; therefore The market has maintained a steady price.

We find that the names of the four forms of the modus ponens are the same as those of the modus tollens. There is indeed an identity between a form of modus ponens and a form of modus tollens having the same name. We may reduce either of them to the other. Let us take the modus ponendo ponens of modus ponens, *viz*. If A then B ; A ; therefore B. If we obvert the contrapositive of the major premise we get, If not B then not A. We may then have the following argument: If not B then not A ; A ; therefore B. This is modus ponendo ponens of modus tollens. Thus we find that the modus ponendo ponens of the modus ponens is essentially the same as the modus ponendo ponens of modus tollens.

Mixed Disjunctive Syllogisms.

Mixed disjunctive syllogisms defined, explained and illustrated.

Mixed disjunctive arguments, like mixed hypothetical arguments, though they are called syllogisms, cannot be regarded as syllogisms proper, since in them there is no middle term. Some logicians distinguish between disjunctive propositions, the form of which is S is not both P and Q, and alternative propositions, the form of which is S is either P or Q. We have not in our treatment laid much emphasis upon this distinction, and have described both forms by the same name, *viz.* disjunctive propositions. Our discussion of the nature of these propositions in the previous book has already given hints as to the nature of disjunctive arguments. According to Keynes, "A **disjunctive** (or **alternative**) **syllogism** may be defined as a formal reasoning in which a **categorical premise** is combined with a **disjunctive** (alternative) **premise** so as to yield a **conclusion** which is **either categorical or** else **disjunctive** (alternative) **with fewer alternants than are contained in the disjunctive premise."** In a mixed disjunctive syllogism, as it is sometimes called, the **major** premise is a **disjunctive** proposition, the **minor** premise is a **categorical** one, the conclusion is either a categorical or a disjunctive proposition. When the **disjunctive** proposition has but **two** alternatives, the **conclusion** is **categorical,** *e.g.* S is either P or Q, S is not P, therefore S is Q; or, Either S is P or S is Q, S is not P, therefore S is Q. When the **alternatives** are **more than two,** the **conclusion** is a **disjunctive** proposition, *e.g.* S is either P or Q or M; S is not P; therefore S is either Q or M. Such a form

of disjunctive proposition is said to be in the modus tollendo ponens, because the minor premise denies one of the alternatives and the conclusion affirms the other alternative or alternatives.) We have in a previous chapter pointed out that where the alternatives of a disjunctive proposition are exhaustive but not exclusive, the modus tollendo ponens is legitimate. If we take it for granted that the alternatives 'intelligent' and 'industrious' are the only alternatives by which we can account for the striking success of a man, say X, we may argue in this way: X is either intelligent or industrious; He is not intelligent; therefore He is industrious. In this case the alternatives are not exclusive, and therefore by affirming one alternative we cannot deny the other, because X may be both intelligent and industrious.

Disjunctive syllogisms in the modus tollendo ponens explained.

A disjunctive syllogism in the modus tollendo ponens may have four forms :—(1) X is either P or Q; X is not P; therefore X is Q; (2) X is either P or not Q; X is not P; ∴ X is not Q; (3) P is either not P or not Q; X is P; ∴ X is not Q; (4) X is either not P or Q; X is P; ∴ X is Q. Each of the above arguments can be reduced to a mixed hypothetical syllogism, and we then find that the arguments 1, 2, 3 and 4 are respectively in modus tollendo ponens, modus tollendo tollens, modus ponendo tollens and modus ponendo ponens. Of the above four forms, the first may be thus reduced to a hypothetical syllogism: If X is not P, it is Q (major); X is not P (minor); ∴ X is Q. We need not reduce the other three forms of disjunctive syllogism to hypothetical ones. Since disjunctive

syllogisms can be reduced to hypothetical ones, which again can be reduced to categorical syllogisms, there is an essential unity between them all. The canon of the above form of the disjunctive syllogism is, To deny one (or more) of a number of alternatives is to affirm the remaining alternative or alternatives. Joseph rightly remarks that the use of a disjunctive argument lies more in what it can establish than in what it can overthrow.

Disjunctive syllogisms in modus ponendo tollens explained.

We have also shown in the previous book that when the alternatives of a disjunctive proposition are exclusive but not exhaustive, we can, by affirming one of the alternatives, deny the other, but by denying one of the alternatives we cannot affirm the other. Thus the proposition, This flower is either green or red, is a disjunction of which the alternatives are exclusive but not exhaustive. With such a disjunctive major premise we may have the following argument: This flower is either green or red; This flower is green; ∴ This flower is not red. Here by denying one of the alternatives we cannot affirm the other, because the flower may be yellow. Such disjunctive syllogisms as the one given above are in the modus ponendo tollens. They also may have four forms, *viz.* (1) S is either P or Q; S is P; ∴ S is not Q; (2) S is either P or not Q; S is P; ∴ S is Q: (3) S is either not P or not Q; S is not P; ∴ S is Q; (4) S is either not P or Q; S is not P; ∴ S is not Q. When the disjunctive arguments are reduced to hypothetical forms, the above forms 1, 2, 3, 4 are found to be in modus ponendo tollens, modus ponendo

ponens, modus tollendo ponens and modus tollendo tollens, respectively. The canon of the disjunctive syllogism in the modus ponendo tollens is, To affirm one member (or more) of any alternative is to deny the other member or members. We may remark that when the alternatives of a disjunctive proposition are both exclusive and exhaustive, as in the case 'This flower is either white or not white', we may by denying one of the alternatives affirm the other, and by affirming one of the alternatives deny the other. With such a disjunctive proposition as the major premise, we may have disjunctive syllogisms both in the modus tollendo ponens and in the modus ponendo tollens.

Dilemma

Dilemma defined and its nature stated.

We have already discussed two forms of what is called the mixed syllogism, and may now consider its remaining form, *viz*. the **Dilemma**, which is more complex than either the hypothetical or the disjunctive argument. If it be true that neither hypothetical nor disjunctive arguments are syllogisms proper, it is also true that dilemmas are not properly speaking syllogisms, though they are so called. A **dilemma** may be defined as a formal argument containing a **premise** in which **two hypothetical propositions** are **conjunctively affirmed,** and a **second premise** which is a **disjunctive proposition** in which the **antecedents** of these **hypotheticals** are **alternatively affirmed, or** their **consequents alternatively denied.** Thus a dilemma has as its major premise two hypothetical propositions having two distinct

antecedents and a common consequent, or two distinct consequents and a common antecedent, or two distinct antecedents and two distinct consequents. The minor premise may either affirm the antecedents or deny the consequents, and the conclusion will affirm the consequent or consequents as the case may be, or deny the antecedent or antecedents as the case may be. When the **conclusion** affirms only **one** consequent or denies **one** antecedent, it is a categorical proposition, and the dilemma is then **simple.** When the **conclusion** affirms **two** alternative consequents or denies two alternative antecedents, the dilemma is then **complex**, and the conclusion is a disjunctive proposition. Further, when the **minor** premise **affirms** the antecedents and the **conclusion** **affirms** the consequent or consequents, the dilemma is said to be **constructive**, and it may be said to be in the modus ponens. But when the **minor** premise **denies** the consequents and the **conclusion denies** the antecedent or antecedents, the dilemma is regarded as **destructive** and may be said to be in modus tollens.

We thus find that the dilemma combines into one argument hypothetical and disjunctive reasoning. The aim of a dilemma is to prove something against an opponent, and it is therefore unpalatable and disagreeable to him. It is therefore defined by Joseph as "a hypothetical argument offering alternatives, and proving something against an opponent in either case." We have a dilemma when the major premise offers two antecedents or two consequents and the minor premise alternatively affirms or denies them respectively. When the major premise

provides three distinct antecedents or three distinct consequents, and the minor premise affirms three alternatives or denies three alternatives respectively, we have what is known as Trilemma. When the alternatives affirmed or denied in the minor premise are four in number, we have Tetralemma, and when they are more than four in number we have what is known as Polylemma. Since the Trilemma, Tetralemma, and Polylemma do not involve any new principle not operative in the dilemma, we need not treat of them separately.

We have already seen that a dilemma may be either simple or complex, and it may also be either constructive or destructive. So in the main there are four forms of the dilemma :—1. Simple constructive.—A **simple constructive dilemma** is one in which the **major** premise, which contains **two hypotheticals,** provides **two** distinct **antecedents** and a **common consequent;** the **minor** premise **alternatively affirms** the **antecedents** ; and the **conclusion affirms** the **consequent.**

Simple constructive dilemma illustrated.

E.g., If A is B, X is Y, and if C is D, X is Y (major premise); Either A is B or C is D (minor premise); therefore X is Y (conclusion). Joseph gives an interesting concrete example of this form : Troops having an impassable river behind them and a deadly enemy in front may be faced with the following dilemma : "If they stand their ground they die—by the sword of the enemy ; if they retreat they die—by the flood ; but they must either stand or retreat ; therefore they must die." Thus we find that whatever alternative of a dilemma is accepted, the result is unpleasant ; hence the saying

'to be on the horns of a dilemma.' An opponent who is faced by a dilemma is between the devil and the deep sea.

II. Complex constructive.—In a **complex constructive dilemma** the **major** premise provides **two** distinct **antecedents** and **two** distinct **consequents**; the **minor** premise **alternatively affirms** the **antecedents**; and the **conclusion alternatively affirms** the **consequents**; *e. g.*, If A is B, X is Y, and if C is D, M is N; Either A is B or C is D; therefore Either X is Y or M is N. The following is a concrete example: "If there is a censorship of the press, abuses which should be exposed will be hushed up; and if there is no censorship, truth will be sacrificed to sensation; but there must either be a censorship or not; therefore either abuses which should be exposed must be hushed up, or truth be sacrificed to sensation" (Joseph).

Complex constructive dilemma illustrated.

It is very difficult to see an escape from this dilemma, as from the previous one.

III. Simple destructive.—A **simple destructive** dilemma is one in which the **major** premise provides a **common antecedent** and **two** distinct **consequents**; the **minor** premise **alternatively denies** the **consequents**; and the **conclusion denies** the **antecedent**; *e. g.* If A is B, X is Y, and if A is B, M is N; Either X is not Y or M is not N; therefore A is not B. The following is a concrete example: "If Homer speaks truth about things divine, the heroes were sons of gods, and did

The simple destructive dilemma illustrated.

many wicked deeds; but either they were not sons of gods, or they did not do wicked deeds; therefore Homer does not speak truth about things divine" (Joseph).

IV. Complex Destructive.—A **complex destructive** dilemma is one in which the **major** premise provides **two** distinct **antecedents** and **two** distinct **consequents**; the **minor** premise **alternatively denies** the **consequents**; and the **conclusion alternatively denies** the **antecedents**; *e. g.* If A is B, X is Y, and if C is D, M is N; Either X is not Y or M is not N; therefore Either A is not B or C is not D. The following is a concrete example:—"If we give our colonies self-government, we shall make them powerful; and if we attempt to control their use of it, we shall make them hostile; But either we ought not to make them powerful, or we ought not to make them hostile; therefore Either we ought not to give them self-government or we ought not to attempt to control their use of it" (Joseph).

The complex destructive dilemma illustrated.

Mansel, and some other logicians following him, hold that destructive dilemmas are always complex and cannot be simple. But our previous discussion shows that destructive dilemmas can legitimately be simple.

Jevons maintains that a dilemmatic argument is more often fallacious than not, because the alternatives are not usually exhaustive. But this view does not appear to be acceptable. A wrong notion persists that the reasoning involved in the dilemma is not sound. But the formal validity of a dilemma cannot be questioned

A dilemmatic argument need not be always false.

if its structure is sound, that is, if its major premise, minor premise and conclusion are what they ought to be according to the definition of the dilemma. A dilemmatic argument may however be materially false, if in the major premise the consequent does not follow from the antecedent, or if the alternatives are not exhaustive A dilemma which is materially false can be rebutted or refuted by a counter-dilemma. Thus the following dilemma—If A is B, X is Y, and if C is D, M is N—can be rebutted by the dilemma, If A is B, M is not N, and if C is D, X is not Y. Only a complex constructive dilemma can be rebutted. The alternatives of a dilemma can be exhaustive only when they are contradictories, but often they are not so. When they are not exhaustive, a man may escape between the horns of the dilemma, that is, between the alternatives. Thus an attempt is made to refute the famous dilemma by which Zeno disproved the existence of motion, which is: If a body moves, it must either move in the place where it is, or in the place where it is not; But it can neither move in the place where it is, nor in the place where it is not; therefore It cannot move. In reply to this dilemma, it is pointed out that a body need not move either where it is or where it is not, but it may move between these two places. But this attempt to refute the dilemma is unsuccessful, because if the body moves at all, it must move either where it is or where it is not.

Three ways of refuting a dilemma, viz. by rebutting it, by escaping between its horns, or by taking the dilemma by its horns.

A third way of refuting a dilemma is to take it by the horns, that is, to show that the consequent or consequents do not follow from the antecedents.

In such a case the major premise can be replaced by another premise which refutes the former, and in which the consequent or consequents follow from the antecedents. If anybody argues thus: If you take milk, you will suffer from indigestion, and if you take bread, you will suffer from indigestion; but You must either take milk or bread; therefore You must suffer from indigestion, we may refute him by the counter-dilemma: If a person takes milk he will be vigorous, and if he takes bread he will be vigorous; He must either take milk or bread; therefore He must be vigorous. This dilemma may also be refuted by pointing out that the person may take fruit instead of taking either milk or bread, and thus there is an escape between the horns of the dilemma.

In the preceding paragraphs we have shown that in order to rebut a dilemma we combine the antecedents of each of the two hypothetical propositions which form the major premise, with the contrary or contradictory of the consequent of the other. We have also remarked that only a complex constructive dilemma can be rebutted. The following is a classical example of how a dilemma is rebutted:—

Some classical examples of the dilemma.

An Athenian mother wanted her son not to enter public life, and advanced the following dilemma to dissuade him: "If you act justly, men will hate you, and if you act unjustly, the gods will hate you; but you must act either justly or unjustly; therefore, public life must lead to your being hated". The son rebutted the above dilemma by the following: "If I act

justly the gods will love me, and if I act unjustly men will love me; therefore, entering public life will make me beloved." It may be pointed out that the two conclusions are not really incompatible, because a public man is always both hated and loved.

We may conclude this topic with two other classical examples of the dilemma. *Litigiosus* is a famous dilemma. Protagoras consented to give lessons to Euathlus in rhetoric, and it was agreed that one-half the fee should be paid at once and the other half when Euathlus won his first case. When Protagoras found that Euathlus engaged in no suit, he sued him and advanced the following dilemma: "If you lose this suit you must pay me by order of the court, and if you gain it you must pay me by our contract." To which Euathlus retorted: "If I lose this suit I am free from payment by our contract, and if I gain it, I am exonerated by the judgment of the court." The best solution of this difficulty is that since Euathlus had won no case up to that time, the judges would decide in his favour. After this Protagoras, when he saw Euathlus had won a case, might sue him again and might reasonably expect that the judges would decide in his favour. Another famous dilemma is known as *Crocodilus*. A crocodile seized a child and told its mother that he would give it back if the mother could say correctly whether he would give it back or not. Fearing that if she said he was going to give it back he would prove her wrong by devouring it, she replied that he would not give it back, and argued: "Now you must give it back; for if my answer is true you must give it back in accordance with your promise, and if the answer is false you must give it back to prevent

its being true." The crocodile replied : "I will not give it back, for if I did, your answer would be false and I should break our agreement; and even if your answer were correct I could not give it back, as that would make your answer false." There seems to be no way out of this dilemma, but the answer would have been more fortunate if the mother had said that he would give the child back, for in that case its restoration would both have made her answer true and have fulfilled the agreement.

CHAPTER V

THE ENTHYMEME, SORITES AND EPICHEIREMA

The Enthymeme

The arguments to be considered in this chapter do not involve any principles other then those considered in connection with the syllogism. We should regard an Enthymeme as a particular way of argument rather than as a new form of argument. The **Enthymeme,** according to Aristotle, is a rhetorical syllogism, as distinguished from the apodcictic, demonstrative, and theoretical syllogism. In an enthymematic argument one of the premises or the conclusion of a syllogism is understood and not expressed. It is therefore a syllogism incompletely stated. So Welton defines an Enthymeme as "A **syllogism abridged** in expression by the **omission** of one of the **constituent propositions.**"

The Enthymeme defined.

In ordinary discourse syllogistic arguments are not usually expressed in full, and the arguments of everyday life are to a large extent enthymematic. Outside the treatises on logic we hardly meet with syllogisms of which the constituent propositions are all expressed. People often take recourse to enthymemes to make a fallacious argument appear true. It is therefore often a very useful means to cover fallacies in reasoning.

Enthymematic arguments common in ordinary discourse.

When the **major premise** of an **enthymeme** is **suppressed**, it is said to be of the **first order** ; when the **minor** premise is **suppressed** it is said to be of the **second order** ; and when the **conclusion** is **omitted**, it is said to be of the **third** order. Every soul is a spiritual substance, therefore No soul is tangible, is an enthymeme of the first order, because here the major premise, No spiritual substances are tangible, has been suppressed. No spiritual substances are tangible, and therefore No soul is tangible, is an enthymeme of the second order, because here the minor premise, Every soul is a spiritual substance, has been suppressed. No spiritual substances are tangible, and Every soul is a spiritual substance, is an enthymeme of the third order, because here the conclusion, No soul is tangible, has been suppressed. Similarly, John is ambitious, therefore he is unhappy, is an enthymeme of the first order ; All ambitious men are unhappy, and therefore John is unhappy, is an enthymeme of the second order ; while All ambitious men are unhappy, and John is ambitious, is an enthymeme of the third order. We need not multiply examples to explain the nature of enthymemes of different orders.

The three orders of enthymemes explained and illustrated.

Enthymemes may consist of hypothetical as well as of categorical propositions. Joseph remarks that "A syllogism, whether expressed in full or as an enthymeme, is a single act of inference ; it may be analysed into premises and conclusion, but not into parts which are themselves acts of inference."

Episyllogistic and Prosyllogistic Trains of Reasoning

A **polysyllogism** or train of syllogisms is a **combination** of a number of **syllogisms,** the **conclusion** of **one** syllogism becoming the **premise** of **another.** Such a train of syllogisms may consist either of categorical propositions or of hypothetical ones. Thus according to Keynes, "A chain of syllogisms, that is, a series of syllogisms so linked together that the conclusion of one becomes a premise of another, is called a polysyllogism". In a polysyllogism, that syllogism the conclusion of which is a premise of another syllogism is called, in relation to the latter, a **prosyllogism** ; while the syllogism which uses the conclusion of another syllogism as one of its premises is called, in relation to that other syllogism, an **episyllogism.** A syllogism which in relation to one syllogism is a prosyllogism may be an episyllogism in relation to another. We may illustrate the matter by an example : A is C, B is A, ∴ B is C ; B is C, D is B, ∴ D is C ; D is C, E is D, ∴ E is C. In this train, B is C, D is B, ∴ D is C is a prosyllogism in relation to the syllogism D is C, E is D, ∴ E is C, because the latter syllogism uses the conclusion of the former as one of its premises ; but it is an episyllogism in relation to the syllogism A is C. B is A, ∴ B is C, because one of its premises is the conclusion of the latter syllogism. Thus we find that a prosyllogism proves a premise of an episyllogism. The following is a concrete example : All material bodies are subject to decay, Animal bodies are material,

The nature of a polysyllogism or train of syllogisms.

A prosyllogistic train of reasoning illustrated.

∴ Animal bodies are subject to decay; Human bodies are animal bodies, ∴ Human bodies are subject to decay; The body of John is an animal body, ∴ The body of John is subject to decay.

The trains of reasoning given above are called synthetic, episyllogistic or progressive. In a progressive train of syllogisms thought moves from prosyllogism to episyllogism. A train of reasoning is said to be regressive, analytic or prosyllogistic when thought moves from episyllogism to prosyllogism. The following is an example of a prosyllogistic train: All subject nations are unprogressive; India is a subject nation; ∴ India is unprogressive; All those nations the government of which does not express the will of the people are unprogressive; All subject nations are nations the government of which does not express the will of the people; ∴ All subject nations are unprogressive. In this train the syllogism last stated proves a premise of the syllogism stated first. In actual occurrence of thought, both progressive and regressive trains of reasoning are met with. Thought advances from prosyllogism to episyllogism when the most general principle is stated first, and it moves from episyllogism to prosyllogism when the most general principle is stated at the end.

An episyllogistic train of reasoning illustrated.

The Sorites

A **Sorites is a progressive,** episyllogistic or synthetic **train of reasoning** composed of **a number of enthymemes.** Keynes defines it as "a polysyllogism in which all the conclusions are omitted

Sorites defined.

except the final one, the premises being given in such an order that any two successive propositions contain a common term." In a sorites the first syllogism of the series or chain is an enthymeme of the third order, while the last is an enthymeme of either the first or the second order. There are **two forms** of sorites, *viz.* the Aristotelian and the Goclenian, which may be illustrated by the following symbolic examples: Aristotelian sorites—

Sorites illustrated by symbols.

A is B, B is C, C is D, D is E, ∴ A is E; Goclenian sorites—D is E, C is D, B is C, A is B, ∴ A is E. The order of the Goclenian sorites is the reverse of the order of the Aristotelian. In the Aristotelian sorites the term which

The Aristotelian and Goclenian sorites compared.

is common to any two successive premises occurs as predicate in the premise stated first and as subject in that which follows; while in the Goclenian sorites the term which is common to any two successive premises occurs first as subject and then as predicate. Further, in the Aristotelian sorites the premise which is stated first in the series contains the subject of the conclusion, and the premise stated last the predicate of the conclusion; while in the Goclenian sorites the first premise of the chain contains the predicate of the conclusion, and the last the subject. It will be found immediately, when we fully draw out the Aristotelian

The fully developed forms of the sorites illustrated above.

and Goclenian sorites, that in the former the premise stated first and all the suppressed premises are minor premises, while in the latter the premise stated first and all the suppressed premises are major

premises. We may now develop the two forms of sorites illustrated above, to make clear what we have stated in the preceding lines. Aristotelian sorites—B is C, A is B, ∴ A is C; C is D, A is C, ∴ A is D; D is E, A is D, ∴ A is E. Goclenian sorites :—D is E, C is D, ∴ C is E; C is E, B is C, ∴ B is E; B is E, A is B, ∴ A is E.

Some concrete examples.

We may now provide a concrete example to illustrate the sorites. The following example is given by Aristotle: Action is that in which happiness lies; what contains happiness is the end and aim: the end and aim is what is highest; therefore action is what is highest. The above example gives us the Aristotelian form of sorites, and we may obtain the Goclenian form by reversing the order of the premises, as follows :—The end and aim is what is highest; what contains happiness is the end and aim; action is that in which happiness lies; therefore action is what is highest. The following statement of St. Paul in the *Romans* provides an example of the sorites: "For whom he did foreknow, he also did predestinate to be conformed to the image of his son.. ...Moreover whom he did predestinate, them he also called; and whom he called, them he also justified; and whom he justified, them he also glorified." The above is an Aristotelian sorites, and if the premises are reversed as before, we may have the form of the Goclenian sorites.

There may be sorites consisting of hypothetical propositions. Welton gives the two following examples :— "If any man is avaricious he is intent on increasing his

The sorites consisting of hypothetical propositions illustrated.

wealth; if he is so intent, he is discontented; if he is discontented, he is unhappy; therefore if any man is avaricious, that man is unhappy." In this case all the constituent syllogisms are pure hypothetical ones, while in the following example the last syllogism is a mixed hypothetical syllogism: "If the soul thinks, it is active; if it is active, it has strength; if it has strength, it is a substance; now the soul thinks; therefore the soul is a substance." We may here remark that both the Aristotelian and the Goclenian sorites are progressive, and the Goclenian sorites should not be regarded as a regressive train of reasoning, as some logicians suppose it to be. According to Hamilton the Aristotelian sorites is an argument in comprehension, while the Goclenian sorites is an argument in extension.

Rules of the Aristotelian sorites stated.

We may now state the **rules** of **sorites.** The two **rules** of the **Aristotelian** sorites are: (1) only **one** premise can be **negative,** and if one is negative, it must be the **last**; (2) only **one** premise can be **particular,** and if one is particular, it must be the **first.** The above rules may be proved thus:—If two premises of a sorites are negative, then we shall come across a syllogism in the series consisting of two negative premises, which we have found is not allowed by a general rule of the syllogism. Again if one of the premises of a sorites is negative, the conclusion must be negative, and the predicate of the conclusion will therefore be distributed. So the term which is the predicate of the conclusion has to be distributed in the premise

in which it occurs. We have found that in the Aristotelian sorites the predicate of the conclusion is the predicate of the premise stated last, and it can be distributed if that premise is negative. So it is proved that if one premise is negative, it must be the last in the Aristotelian sorites. Further, if more than one premise is particular in a sorites, we shall come across a syllogism in the series with two particul r premises, which is not allowed by a general rule of the syllogism. Further, if the last premise of an Aristotelian sorites be particular instead of the first, then there will be the fallacy of undistributed middle, because the last premise is the major premise, of which the middle term is the subject, and it can be distributed only when it is universal. The other premise, which is suppressed, is the minor premise, and has the middle term as its predicate, and this premise being affirmative (since the last premise alone in the Aristotelian sorites can be negative) cannot distribute the middle term.

The following are the two **rules** of the **Goclenian** sorites :—(1) only **one** premise can be **negative,** and if one is negative, it must be the **first** ; (2) only **one** premise can be **particular,** and if one is particular, it must be the **last.** If more than one premise in this form of sorites is negative, we have a syllogism with two negative premises, as in the case of the Aristotelian sorites, which is not allowed by a general rule of the syllogism. If one of the premises is negative, the conclusion will be negative, and the predicate of the conclusion will be dis-

The rules of the Goclenian sorites stated and proved.

tributed. The term which is the predicate of the conclusion has therefore to be distributed in the premise in which it occurs. We have found that in the Goclenian sorites the first premise in the series has for its predicate the term which is the predicate of the conclusion. So the first premise must be negative, if any premise in the Goclenian sorites is to be negative, in order to distribute the required term. If more than one premise is particular in this form of sorites, we shall come across a syllogism in the syllogistic train consisting of two particular premises, as in the case of Aristotelian sorites; but we know that this is not allowed, by a general rule of the syllogism. If one of the premises is particular in the Goclenian sorites, it must be the last, which, we have found, cannot be negative and is the minor premise of the last syllogism. The middle term in this premise, being the predicate of an affirmative proposition, cannot be distributed. Therefore it must be distributed in the suppressed major premise. The middle term is the subject of this premise, and can be distributed only if it is universal. Therefore if any premise in the Goclenian sorites be particular, it must be the last, otherwise there will be the fallacy of undistributed middle.

The examples of sorites which we have so far given consist of syllogisms all of which are in the first figure. According to Mill, there may be a sorites in which the last or the first syllogism is either in the second or in the third figure. He gives the following example of a sorites in which the last syllogism is in figure 2: A is B, B is C, C is D, D is E, F is not E, ∴ A is not F. All the premises of the above are supposed to be universal. Keynes

however points out that there may be sorites in the second or in the third figure. A sorites in the second figure consists of syllogisms each of which is in the mood Baroco, and consequently the sorites may be said to be in the mood Baroco. He gives the following example: Some A is not B, Every C is B, ∴ Some A is not C; Every D is C, ∴ Some A is not D; Every E is D, ∴ Some A is not E. Keynes shows that when a sorites is in the third figure, its constituent syllogisms are in the mood Bocardo, and consequently the sorites is in the same mood. He gives the following example to illustrate this: Some D is not E, Every D is C, ∴ Some C is not E; Every C is B, ∴ Some B is not E; Every B is A, ∴ Some A is not E. The above examples are undeniably examples of sorites, but we must remember that the rules of sorites given in the previous paragraph do not apply to them. These rules apply only to those sorites the constituent syllogisms of which are in the first figure.

Keynes proves that sorites in the second and in the third figure are possible.

The Epicheirema

"An **Epicheirema** is a **regressive** chain of reasoning abridged by the **omission** of **one** of the **premises** of **each prosyllogism**" (Welton). It is therefore a polysyllogism with one or more prosyllogisms briefly indicated only. It being a regressive, analytic or prosyllogistic train of reasoning, the movement of thought in the epicheirema is from episyllogism to prosyllogism. The epicheirema may be

The Epicheirema defined and its four forms explained and illustrated.

either **single** or **double,** and it may be either **simple** or **complex.** So it may have four forms, *viz.* (1) simple single, (2) simple double, (3) complex single, (4) complex double. We may illustrate these forms by symbolic examples :—

(1) **Simple single** :—Every M is P because it is X ; Every S is M ; Therefore S is P. This is single because only the major premise of the argument is proved by a prosyllogism, one premise of which is suppressed. If we fully express the prosyllogism which proves the major premise of the episyllogism, we find it to be—Every X is P (suppressed major premise) ; Every M is X ; therefore Every M is P. Thus we find that the prosyllogism which in this case proves one of the premises of the episyllogism, is an enthymeme of the first order. Let us take another example :—M is P because X is ; S is M ; ∴ S is P. In this case, if the prosyllogism, which is an enthymeme, is fully developed, we have the argument: Every X is P (major premise) ; Every M is X (suppressed minor premise) ; ∴ Every M is P. In this case the prosyllogism which proves one of the premises of the episyllogism is an enthymeme of the second order.

(2) **Simple double** :—The following are two symbolic examples of this form of the epicheirema : (*a*) Every M is P, because it is X ; Every S is M, because it is Y ; therefore Every S is P. (*b*) Every M is P, because every X is ; Every S is M, because every Y is ; therefore Every S is P. In the first example, the prosyllogisms which prove the two premises of the episyllogism are enthymemes of the first order, while in the second example they are

enthymemes of the second order. An epicheirema is double when both the premises of the episyllogism are proved by enthymemes.

(3) **Complex single** :—An epicheirema is complex when its premise is proved by an enthymeme which again is proved by another enthymeme. If only one of the premises is so proved, we have the single complex epicheirema, and if both the premises are so proved we have the double complex epicheirema. The following is an example of a single complex epicheirema :—Every M is P, because it is X, and every X is Y ; Every S is M ; therefore Every S is P.

(4) **Complex double** :—The following is an example of the double complex epicheirema: Every M is P, because it is X, and every X is Y ; Every S is M because it is N, and every N is O ; therefore Every S is P.

Some concrete examples.

We may provide a concrete example of the simple double epicheirema : All ambitious men are unhappy because they can never satisfy their desires, and all statesmen are ambitious because they always want to be more powerful ; therefore all statesmen are unhappy. We need not provide many more concrete examples to illustrate the epicheirema. We may conclude with one further concrete example of it given by Joseph : "Those who have no occupation have nothing to interest themselves in, and therefore are unhappy ; for men with nothing in which to interest themselves are always unhappy, since happiness depends on the success with which we advance the objects in which we are interested ; and so wealth is no

guarantee of happiness." Here the central syllogism is: All who have nothing in which to interest themselves are unhappy; Those who have no occupation have nothing in which to interest themselves; therefore those who have no occupation are unhappy.

CHAPTER VI

FUNCTION, VALIDITY AND RANGE OF THE SYLLOGISM

Problems to be discussed in this chapter.

In this chapter we shall consider whether syllogistic reasoning is useful, whether it involves the fallacy of petitio principii, and what is its scope; and lastly we shall briefly indicate the relation between the form and matter of thought, that is, the general nature of the relation between deduction and induction. We shall be able to determine the problem of the value and validity of the syllogism if we understand clearly its function. For a long time it has been argued that the syllogism is either useless or fallacious. Mill puts the matter thus: "If all the facts of the major premise of any syllogism have been examined, the syllogism is needless; and if some of them have not been examined, it is a petitio principii. But either all have been examined, or some have not. Therefore the syllogism is either useless or fallacious."

The syllogism cannot be regarded as useless.

We must here remember that every useful and valid inference must satisfy two conditions, *viz.* (1) it must contribute to the advancement of knowledge, that is, the conclusion must be different from the premises; and yet (2) it must follow necessarily from the premises. Thus the paradox of inference is that the

conclusion arrived at by means of it must be within the premises and yet outside them, that is, it must go beyond the premises though it is implied by them. Does the syllogistic inference satisfy these two conditions? If it does not, it is certainly valueless and invalid. In every syllogism there is a universal proposition, and we have seen that the mood Barbara is the ideal type of syllogism. It is held by the opponents of syllogism that in it the conclusion does not go beyond the premises, and that therefore it is valueless. Let us now see what really is the function of syllogism by taking an example of it and asking whether it is useful. All material bodies gravitate, This stone is a material body, ∴ it gravitates, is a perfect syllogism in the mood Barbara. The opponents of the syllogism hold that if we know that all material bodies gravitate, we also know that this stone gravitates; thus the conclusion does not provide any new information or go beyond the premises, and is therefore valueless and not, properly speaking, an inference. Is this contention true? It appears that the opponents of the syllogism are wrong. The major premise of a syllogism need not be an enumerative universal, that is, it may not be established by observing all the instances comprehended under it. In the illustration given above, the major premise, All material bodies gravitate, is a true abstract universal, because it has been established on the basis of some universal principle after observing only a few instances of material bodies gravitating. Thus, though we are aware of the major premise of the syllogism, the conclusion drawn from it may really provide some new information to the enquiring mind, and in this sense it

goes beyond the premises, though it necessarily follows from them.

Inference is a movement of thought, and its claim to value depends upon subjective, not upon objective, conditions. We have seen that even in immediate inference there is a movement of thought, and it is therefore inference proper. Mere verbal novelty is not sufficient for inference. If we pass from the proposition that George the Fifth is the king of England to the proposition that George the Fifth is England's king, there is no inference, since there is no movement of thought. But in syllogistic reasoning this is not so. The novelty that is required by syllogistic reasoning is subjective novelty, as Keynes puts it, not objective novelty or mere verbal novelty. Let us take the syllogism, All ruminants are herbivorous, Camels are ruminants, ∴ Camels are herbivorous. Here we may be aware of the major premise without being aware of the conclusion, though the former implies the latter; and this is certainly a case of inference proper. Therefore the contention that the syllogism is useless cannot be established. We may also point out that in a syllogism we do not draw a conclusion from the major premise alone, but the minor premise is also necessary. This is recognised both by the dictum of Aristotle and by the dictum nota notæ, as we pointed out in our discussion of the syllogism. If this is so, that is, if the minor premise is an indispensable necessity of the syllogistic argument, there is an additional ground to reject the contention that it is useless, since the conclusion is not drawn from the major premise alone.

Mill's view of the syllogism examined.

Let us now examine Mill's contention that all arguments are from particulars to particulars. He maintains that syllogistic arguments are not, properly speaking, cases of inference. They are simply interpretative. Our arguments are always free from the necessity of taking recourse to the universal. We argue that John is mortal, James is mortal, Philip is mortal etc., therefore the Duke of Wellington is mortal. We need not infer the mortality of the Duke of Wellington from the premise All men are mortal. The universal proposition of a syllogism is nothing but a memorandum of observed facts. We may no doubt argue in this way that John is mortal, James is mortal, Philip is mortal etc., therefore All men are mortal; and the Duke of Wellington is a man, and therefore he is mortal. But such a process of thought is not necessary to arrive at this conclusion, because we can pass directly from the observed facts to the ultimate conclusion without taking the help of the universal proposition All men are mortal. Thus Mill writes, "I cannot perceive why it should be impossible to journey from one place to another unless we march up a hill and then march down again." "Not only may we reason from particulars to particulars without passing through generals, but we perpetually do so reason." "The child who, having burnt his fingers, avoids to thrust them again into the fire, has reasoned or inferred, though he has never thought of the general maxim, Fire burns." But Bradley rightly points out that no inference is possible without the help of some universal element. Even in induction, which Mill advocates so vehemently, it is impossible to establish a valid conclusion

without perceiving the universal connection between the subject of inference and the inferred property. Even when Mill's village matron finds out what the illness of a neighbour's child is from the illness of her own child which she observed in the past, she can correctly do so only if she sees the universal connection between the illness and its symptoms. Only in an analogical argument do we pass from the particular to the particular without the help of any universal element. But such arguments, as we shall later find, are more often fallacious than not.

Since it is impossible to do away with the universal element in reasoning, it is also impossible to dispense with the need of the syllogism. If the syllogistic argument is nothing but the interpretation of the major premise in the conclusion, then even geometrical inference must be regarded as merely interpretative and not as reasoning proper. But in the deductive reasonings of geometry, though conclusions necessarily follow from self-evident axioms and postulates, it cannot be contended that they are apprehended as soon as the axioms and postulates are known. Even if we know the latter, we are not aware of many of the conclusions that are established from them. We have already shown that there is a real advancement of knowledge in syllogistic arguments. By means of syllogisms the laws of Kepler are extended to newly discovered planets and satellites, which had not been observed when the laws were established. Even Mill admits that the argument, All men are mortal, the Duke of Wellington is a man, therefore the Duke of Wellington is mortal, is an argument the conclusion of which is not

apprehended as soon as the major premise is given, since when he was writing his *Logic* the Duke of Wellington was still alive. Even Bradley admits that three-fourths of our reasonings are syllogistic. Thus we find that the syllogism is not useless, for the conclusion is not obviously apprehended as soon as we apprehend the major premise, and the minor premise is indispensable. Only when the major premise of a syllogism is an enumerative universal, that is, is established by observing all the instances comprehended under it, can it be regarded as not very useful. We shall find, when we discuss the validity of induction, that the general proposition established by induction is only hypothetical, and can be verified and proved only when tested by syllogistic reasoning.

The conclusion to which our arguments lead.

The above considerations will now enable us to establish that the syllogistic argument is not fallacious. The charge, that the syllogism involves the fallacy of petitio principii, is of considerable antiquity. It was advanced by Sextus Empiricus in the second century A. D., and in modern times by Mill and others. Sextus Empiricus held that in every syllogistic reasoning the truth of the conclusion is assumed in the truth of the major premise and therefore the fallacy of petitio principii is committed. The fallacy of petitio principii occurs if the premise from which a conclusion is deduced is itself proved by evidence which includes the conclusion. Sextus Empiricus supposed that the major premise of the

The arguments of Sextus Empiricus and Mill as to why syllogistic reasonings involve the fallacy of petitio principii.

syllogism is always an enumerative universal, and that therefore the conclusion cannot go beyond it, as in the syllogism, All the apostles were Jews, Peter was an apostle, therefore Peter was a Jew. It is undeniable that in such a reasoning the conclusion is required to prove the major premise, and therefore is assumed in the major premise. But here also, we must remember, the conclusion follows not merely from the major premise, but from two premises, the major and the minor. And Sextus Empiricus is wrong in supposing that the universal proposition in the syllogism is always an enumerative universal. We have already found that a syllogism which is of value requires a major premise which is a true abstract universal.

The charge that the syllogism is fallacious refuted.

Mill and his followers also regarded syllogisms as unnecessary and fallacious. Mill does not seem clear as to the meaning of the fallacy of petitio principii. Does he mean that in a syllogism, if the conclusion is false, the premises (provided the process of reasoning is correct) are also false? This is not the proper meaning of the fallacy of petitio principii. In this sense not only the syllogism but every other form of demonstration, geometrical or other, involves petitio principii, since in every demonstration, if the conclusion is false, the premises cannot be true. But the charge of petitio principii can only be brought in a true sense against the syllogistic argument, if it can be shown that in syllogism the conclusion is required as evidence to prove the major premise and is thus assumed in that premise. But we have already seen good reason to conclude that this contention cannot be granted.

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Keynes and Johnson rightly remark that the validity of syllogistic reasoning rests upon subjective or epistemic conditions, not upon objective or constitutive conditions. Objectively viewed, the syllogism can be regarded as fallacious, because the objects to which the major premise refers include the object or objects to which the conclusion refers ; and from this point of view Mill regarded the syllogism as involving the fallacy of petitio principii. But we have already pointed out that the essence of inference is movement of thought, and if we do not know the conclusion as soon as we know the premises, and the conclusion imparts some new information, there is inference proper. If so, the epistemic or subjective validity of the syllogism cannot be questioned, and the epistemic factor of inference, which is concerned with what we happen to know, rather than the constitutive factor, which is concerned with what is thought about and is independent of us, is what determines the validity of inference.

Summary of the results reached.

Welton therefore sums up as follows the arguments required to prove the validity of syllogistic reasoning : "The major is essentialy not a mere summation of observed instances ; the minor is a necessary part of every syllogism ; it is possible to accept the premises without drawing the conclusion, and hence to make progress in knowledge by means of syllogism ; and the fact of inference depends on the rigidity of the proof, not on its novelty." A syllogism does involve the fallacy of petitio principii if its major premise is an enumerative universal. Again, if a syllogism is proved by another

syllogism, and this again by a third, and so on, as in the case of a train of reasoning, there is an infinite regress. and in such a case the fallacy of petitio principii occurs, because the syllogism which proves another syllogism itself requires proof. The syllogism may also be regarded as fallacious if viewed objectively; but in deductive reasoning, as we have found, subjective validity is sufficient.

Range and limitations of the syllogism. Attempt to reduce all inferences to the syllogistic form is not tenable.

We may now consider the range and limitations of the syllogism. It has been claimed by Whately, Bowen, Mansel and others that the syllogism is the only type of mediate inference and that every mediate inference can be syllogistically expressed. Whately attempted to reduce inductive reasoning to the syllogistic form. Mansel attempted to reduce the argumentum a fortiori to the syllogism. Thus he reduces the argument, A is greater than B, B is greater than C, ∴ A is greater than C, to the following syllogism: Whatever is greater than a greater than C is greater than C; A is greater than a greater than C; ∴ A is greater than C. But in this argument the major premise assumes the conclusion, and B, which occurs in the conclusion, does not occur in the premises; therefore this is not a syllogism proper. Syllogism is only one of the various forms of mediate inference and should not be regarded as the only type.

We have seen that the relation from which the syllogistic inference is drawn is that of subject and predicate;

but from other relations we can draw inferences which cannot be regarded as syllogistic. Thus Welton says, "The syllogism deals only with propositions which express the relation of subject and attribute, and inferences from other relations, though they may be perfectly valid, not only are not made syllogistically, but cannot be satisfactorily expressed in that form". We require a logic of relatives to find out all the relations from which inferences may be drawn, but since relations are vague, it is difficult to elaborate such a logic in its perfect form. Bradley says that the principles of synthesis of relations, upon which all inferences depend, are as many as there are categories, but he mentions five main types of these principles. These are :—

Bradley's consideration of the principles of synthesis of relations shows that the syllogism is not the only type of mediate inference.

(1) Synthesis of subject and attribute, as illustrated in the case, All men are mortal, Socrates is a man, ∴ Socrates is mortal. All syllogistic inference rests upon this principle of synthesis.

(2) Synthesis of identity, as illustrated in the case, A is the brother of B, B of C, and C is the sister of D, ∴ A is the brother of D.

(3) Synthesis of degree, as illustrated in the case, A is hotter than B, B is hotter than C, ∴ A is hotter than C; or, A is greater than B, B than C, ∴ A than C. This principle of synthesis is the basis or ground of the argumentum a fortiori.

(4) Synthesis of time, as illustrated in the case, A is

before B, and B before C, ∴ A before C ; or, A is after B, B is contemporary with C, ∴ A is after C.

(5) Synthesis of space, as illustrated in the case, A is north of B, and B west of C, ∴ C is south-east of A.

Though this list of the principles of synthesis of relations is not exhaustive, it throws much light upon the problem of the different forms of mediate inference. Of these principles, the syllogistic inference rests upon the first alone. The inferences illustrated in the other four cases are not syllogistic, and have no universal major premise. Thus we find that the syllogism is not the only form of mediate inference, and not even the only form of deductive mediate inference, for all the inferences illustrated above are deductive, though only the first is syllogistic. Though this is true, it is undeniable that the syllogism is the most important, the most universal, and the most accurate form of inference, and the major portion of our inferences are syllogistic. Though the syllogism is limited in range, it is perfect as far as it goes.

The syllogism not the only type of deductive mediate inference, but by far the most important.

[Mr. Russell* has thrown much light upon the problem of the logic of relatives. It is owing to his work that we can now provide a list of the most important relations which are the ground of inference :—

1. Subject and predicate relation.—This relation, as we know, is the ground of syllogistic inference.

* The portion within square brackets may be omitted by Intermediate students.

Russell's classification of relations from which inferences may be drawn.

2. Transitive relation.—This also can yield valid conclusions. "A transitive relation is such that if it relates one term to a second term and this second term to a third term, then it must relate the first to the third term". *E.g.*, If A is younger than B, and B is younger than C, then A is younger than C.

3. "An intransitive relation is such that if it relates one term to a second and this second to a third, it cannot relate the first term to the third. Thus if A is the father of B and B is the father of C, A cannot be the father of C."

4. "A non-transitive relation is such that it may or may not be transitive. Thus A, who is a friend of B who is a friend of C, may or may not be a friend of C."

5. Symmetrical relations.—"A symmetrical relation is such that if it relates one term, A, to another, B, it also relates B to A;" *e.g.* 'equals', 'is identical with.' If A equals, or is identical with, B, then B equals, or is identical with, A.

6. Asymmetrical relations.—"An asymmetrical relation is such that if it relates A to B it cannot relate B to A;" thus 'after' is an asymmetrical relation. If A is after B, then B is not after A.

7. Non-symmetrical relations.—"A non-symmetrical relation is such that if it relates A to B, it may or may not relate B to A;" *e.g.* 'hate'. If X hates Y, Y may or may not hate X.

8. Connected relation.—"A connected relation is such that given any two of the terms which it relates,

it either relates the first to the second or the second to the first, and it may relate both the first to the second and the second to the first. Thus among the points on a line, of any given two, one must be to the left of the other, or this must be to the left of the one." "Relations which are at once transitive, asymmetrical and connected are of special importance in mathematical reasoning, since they give rise to series." Transitiveness is a relation which secures the validity of non-syllogistic reasoning. Some relations occurring in non-syllogistic arguments are asymmetrical as well as transitive. This is evident in the a fortiori reasoning.]

Dogmatism either of formal logicians or of the advocates of material logic is not tenable.

The progress of knowledge depends upon correct inference, and we know that the two main types of inference are syllogism and induction. Some logicians, such as Whately, Mansel and others, supposed, as we have found, that the syllogism is the only type of valid mediate inference, and they refused to regard induction as an independent form of it. On the other hand Mill and his followers minimised the importance of syllogistic reasoning; they supposed that inference is always from the particular to the particular, that the major premise of the syllogism is a mere memorandum which registers observed experiments, and that the passage from the general to the particular in the syllogism is not inference but mere interpretation of what has been observed. Our discussion has shown that neither syllogistic inference nor inductive inference can be ignored by thought. Both types are necessary for knowledge. The major premise of the syllo-

gism, if it is not a self-evident axiom, must express a belief which has been proved by induction, if the syllogistic reasoning is to be valid. Similarly, if induction is to prove or discover some law which is true and useful, it must not depend simply upon observation of facts, but it should also be formally valid and its conclusion verified by syllogistic reasoning. The study of the formal principles of thought becomes barren if they are not the forms of things as well. Similarly the study of particular facts cannot contribute to the advancement of knowledge unless the forms which exist in them are also studied at the same time. So it is true to say that form without matter is empty, and matter without form is barren. Sciences study different branches of phenomena, and when we have discovered all the main branches of possible knowledge, we can study the forms of knowledge even when our knowledge is incomplete in extent. But as Joseph rightly remarks, "Validity of form is a thing worth studying, not only for its own sake, but in some degree lest we infringe it; yet it is psychologically possible, by studying it too much and too exclusively, to become distracted from due care about truth of fact." He also says, "If this is true, we may say on the one hand, that no study of the nature of inference can be adequate which treats it as an operation performed with symbols, or one intelligible at any rate when we work merely with symbols." Those logicians who reduce the study of logic to the study of symbols make it a barren science, and such a symbolic must be regarded as incapa-

Form and matter of thought cannot be divorced from each other.

The study of formal logic is useful by itself but it should not be too formal.

ble of contributing much to the advancement of knowledge. Modern logicians rebelled against the dogmatic logic of the Schoolmen because it did not recognise the importance of the study of nature. From the time of the Reformation and Renaissance men began to study nature with greater attention. "The mandate issued to the age of Plato and Aristotle was, Bring your beliefs into harmony with one another; the mandate of the mediaeval spirit was, Bring your beliefs into harmony with dogma; the mandate of the new spirit which rebelled against the authority of the Church was, Bring your beliefs into harmony with fact." But even ancient and mediaeval logicians recognised that the beliefs and dogmas required by knowledge should be consistent with facts. The premises required by knowledge must be true, that is, consistent with experience, and we must also know how they can be acquired. We do argue from accepted beliefs and opinions provided by experts, as in deduction, and such a method of inference is very useful and necessary for the progress of knowledge, provided there are enquiring spirits to question and examine the validity of the accepted opinions. Induction performs this function, and it is, therefore, absolutely necessary for the proof of the validity of the universal proposition or propositions from which syllogistic reasoning infers. Syllogism, therefore, without the aid of induction cannot establish conclusions which are ultimately valid, that is, consistent with the content of reality.

The old and the modern spirit of logic.

Induction proves the universal premises used by syllogism and therefore the former is the corrective of the latter. The ultimate validity of syllogism depends upon its universal premise being proved by prior induction.

CHAPTER VII

FORMAL FALLACIES

The meaning of fallacy.

The term fallacy is used in different senses. A false statement is often said to be a fallacy. Thus we say that the statement that men walk on their heads is fallacious. It is also held that a false belief is a fallacious belief. Thus to believe in ghosts is regarded as a fallacious belief. Even an ungrammatical sentence, *e.g.* Five men is coming, is said to be a fallacious sentence. But in logic the term fallacy should be used to denote an argument which is really false, though it appears to be true. Properly speaking, a false argument alone should be regarded as a fallacy. A false belief or a false statement should not be so regarded. According to Joseph, "A **fallacy** is an **argument** which **appears to be conclusive when it is not**; and the chief use of studying fallacies must be that we may learn to avoid them." The fallacy is also defined as "a violation of logical principle disguised under a show of validity." It is therefore clear that not every confusion of thought or prejudice should be regarded as a fallacy.

Many logicians are of opinion that fallacies need not be studied as a distinct subject in connection with logic. They hold that the violations of grammatical rules are not studied separately, and in the same way the violations of logical principles need not be studied separately. It is true that a knowledge of the principles of logic enables

The use of the study of fallacies.

us to some extent to detect fallacies arising from their violation, and that a study of fallacies does not enable us to avoid errors in reasoning. It is true that a psychologist can well understand the principles of the operations of the mind only when he studies both abnormal and normal mental processes, and a physiologist can gain a clear understanding of the conditions of health only by studying the principles both of diseases and of health. But a logician cannot acquire insight into the principles of valid thinking by studying fallacies. Though this is true, it cannot be denied that we can know a thing completely only by knowing its opposite. So a study of the transgressions of logical principles along with the study of the principles themselves may enable us to see more clearly what the principles of valid thinking are. Therefore the study of fallacies cannot be regarded as itself fallacious or useless.

Sources of fallacies.

According to Mill there are two main **sources** of **fallacies**, *viz.* **moral** and **intellectual.** Fallacies are often committed as a result of passion, indolence, prejudice etc.; these are moral sources of fallacies. Again, men commit fallacies because they sometimes fail to think rightly. Fallacies which are due to the failure to think accurately are those which have an intellectual source.

It is very difficult to classify fallacies. Various classifications are given but none of them is scientific. Aristotle classifies all fallacies under two main heads, *viz.* (1) fallacies **in dictione,** under which he includes Equi-

Different classifications of fallacies.

vocation, Amphiboly, Composition, Division, Accent, and Figure of Speech; and (2) fallacies **extra dictionem,** under which he includes Accident, Secundum quid, Ignoratio elenchi, Petitio principii, Non causa pro causa, Consequent, and Many Questions. The former class of fallacies, but not the latter, has its source in the ambiguity of language. Whately classifies fallacies under two main heads, *viz.* (1) logical or formal, which he subdivides into (*a*) purely logical, (*b*) semi-logical, and (2) material. Mill gives the following classification:

- Fallacies
  - of simple Inspection ...... 1. Fallacies a priori.
  - of Inference
    - From evidence distinctly conceived.
      - Inductive fallacies
        - 2. Fallacies of observation.
        - 3. Fallacies of generalisation.
      - Deductive fallacies
        - 4. Fallacies of ratiocination.
    - From evidence indistinctly conceived ...... 5. Fallacies of confusion.

Other logicians classify fallacies under three main heads, *viz.* (1) Non-logical or material fallacies, which are subdivided into (*a*) Premise unduly assumed (petitio principii), (*b*) Irrelevant conclusion or Ignoratio elenchi; (2) Non-inferential logical fallacies, which include fallacies of definition, division, classification, etc.; (3) Inferential fallacies, which are either (*a*) inductive or (*b*) deductive. Deductive inferential fallacies are again

subdivided into (i) Formal fallacies and (ii) Semi-logical fallacies. We shall not treat of non-logical or material fallacies and inductive inferential fallacies in this volume. Other fallacies, which are connected with the problems treated of in this volume, will be discussed under three main heads, (1) **Non-inferential logical** fallacies, (2) **Inferential formal** fallacies, (3) **Semi-logical** fallacies. This classification, though convenient, does not claim to be a scientific one.

Non-inferential Logical Fallacies

In consequence of failure to understand the meaning of terms clearly, we often use concepts which are vague.

Vague Conception.

If a term is clearly defined and understood, we can get rid of fallacies incidental to **vague conception.** Thus the term 'infinite' is used to mean 'finite at a very extended limit,' though the term 'infinite' really means 'that which is limitless.' Similarly the term 'eternal' is used to mean 'enduring throughout a very long time.' But the term 'eternal' really connotes existence out of time.

Fallacies incidental to the use of **inconsistent terms** occur when incompatible words are combined into terms.

Inconsistent terms and propositions.

These fallacies also occur because we often fail to see the meaning of words clearly and distinctly. Thus such terms as 'indivisible matter,' 'sweet sorrow,' 'circular straight line,' 'pleasant anger,' 'angular happiness' etc. are inconsistent terms. The use of such terms should be avoided in logic. Again, sometimes we commit the fallacy of **inconsistent propositions,** when the subject

and the predicate which are combined in the proposition are inconsistent terms. Thus the propositions 'Man is immortal,' 'Every rule has an exception,' 'Mind is visible,' 'Truth is consistent falsehood,' 'Misery is desirable,' are inconsistent. The proposition 'Epimenides the Cretan says that all Cretans are liars,' is also self-contradictory or inconsistent, because it means that Epimenides can only speak the truth if he lies, and lies if he speaks the truth. We use inconsistent propositions only when the meaning of the terms of a proposition is not well defined and understood.

Fallacies arising from violation of the rules of Definition, Division and Classification.

We have already discussed, in the chapters on Definition, Division and Classification, the fallacies which arise from the violation of the rules of definition, division and classification. So these fallacies need not again be treated of here. Students will find them clearly and fully discussed in the chapters mentioned. The **fallacies of definition** are redundant definition, too wide definition, too narrow definition, obscure definition, tautologous circular definition, negative definition and figurative definition. The **fallacies of division and classification** are cross or overlapping division, too wide division, too narrow division, metaphysical division, physical partition, artificial classification, etc. These fallacies, though logical, are not fallacies of inference.

Inferential Formal Fallacies

Under this head are included those fallacies which arise from the violation of the rules of opposition, of

Fallacies to be treated in this section.

eductions, of syllogisms, of mixed syllogisms, of sorites and of epicheirema. These fallacies have been explained clearly in connection with the discussion of the topics mentioned. Fallacies incidental to **opposition** occur when the rules of inference by opposition are violated. When two propositions are in contrary opposition, we can pass from the truth of one to the falsity of the other, but not conversely. Thus it is fallacious to argue that if an A proposition is false, the corresponding E proposition is true. Similarly of two subaltern propositions, when the universal is true, the corresponding particular is true, but it is fallacious to argue that when the universal or subalternans is false, the corresponding particular or subaltern is also false Again, though we can pass from the falsity of the subaltern to the falsity of the subalternans, it is fallacious to argue from the truth of the subaltern to that of the subalternans. Again, of two sub-contrary propositions, if one is false, the other is true, but it is fallacious to argue that when one is true the other is false. The confusion between contrary and contradictory opposition often gives rise to fallacy.

Fallacies incidental to opposition.

Eductions, we have found, are of two kinds, *viz.* material and formal. We have shown how material eductions may lead to fallacies, and that the correctness of material eductions depends upon the knowledge of facts. Students will find that fallacies incident-

Fallacies incidental to material and formal eductions.

al to **material eductions** have been treated in connection with the discussion of the forms of those eductions. We need not repeat them here. Fallacies incidental to **formal eductions** are those which arise from the violation of the rules of conversion, obversion, contraposition and inversion. The commonest of these fallacies consists in simply converting an A or an O proposition. If A is simply converted, or O is converted at all, there is the fallacy of **distribution.** It is a rule of eduction that no term should be distributed in the conclusion if it was not distributed in the premise. If A is simply converted, the predicate term of the original proposition, which is undistributed there, becomes distributed in the conclusion, as the subject of a universal proposition. Again, if O is converted, the subject term of the original proposition, which is undistributed, becomes distributed in the conclusion, as the predicate of a negative proposition. Fallacies incidental to contraposition and inversion also occur when a term which is not distributed in the premise becomes distributed in the conclusion. These fallacies are really fallacies of conversion. Thus if we conclude from No S is P to All not-P is S, there is the fallacy of **contraposition.** To infer from Every S is P that No not-S is P, or from No S is P that Every not-S is P, would be to commit the fallacy of **illicit inversion.** Similarly if we pass from the proposition 'Thought is existent' to the proposition 'What contains no element of thought is non-existent,' we commit the fallacy of illicit inversion. Students may consult the discussion of formal eductions if they wish to understand clearly the fallacies which arise from the violation of the rules of formal eduction.

The fallacies which arise from the violation of the rules of syllogism have been fully discussed in the chapter on syllogism. Here we need only name them. They are—(1) the fallacy of **four terms** (Quaternio Terminorum), (2) the fallacy of **undistributed middle,** (3) the fallacy of **illicit process** of the **major** term or illicit major, (4) the fallacy of **illicit process** of the **minor** term or illicit minor, (5) the fallacy of **negative premises.** Fallacies are **abstract** when they are committed openly, and **concrete** when they are hidden by the language. The fallacy of undistributed middle is very often committed, and should be guarded against.

Fallacies incidental to syllogisms.

Fallacies incidental to mixed syllogisms have been fully discussed in the chapter on mixed syllogisms. Fallacies of **hypothetical** syllogisms are two in number, *viz.* (1) **denying the consequent,** (2) **affirming the antecedent.** The former occurs when by denying the antecedent we deny the consequent, and the latter occurs when by affirming the consequent we affirm the antecedent. These fallacies are called by Aristotle fallacia consequentis.

Fallacies incidental to hypothetical syllogisms.

Fallacies occur in disjunctive syllogisms when from a disjunctive proposition whose alternatives are exclusive but not exhaustive we affirm one of the alternatives by denying the other. Again, when a disjunctive proposition has its alternatives exhaustive but not exclusive, we commit a fallacy if we deny one of them

Fallacies incidental to disjunctive syllogisms.

by affirming the other. (For illustrations see the chapter on Mixed Syllogisms).

Fallacies incidental to dilemmas.

Dilemmatic arguments are fallacious when the alternatives are not exclusive and exhaustive. We have seen, in discussing the dilemma, how it is possible to escape between the horns of a dilemma, to take it by its horns, or to rebut it. We have pointed out in what circumstances dilemmatic arguments are fallacious. (Students are referred to the chapter on Mixed Syllogisms to find the fallacies incidental to the dilemma).

Fallacies incidental to sorites, epicheirema, and enthymemes.

Fallacies also arise from the violation of the rules of sorites and epicheirema. Students are referred to the chapter on trains of syllogism for the rules of sorites and epicheirema, and for the fallacies which may arise from their violation. Fallacies incidental to enthymemes are nothing other than the fallacies of syllogisms. To find out the fallacy in an enthymematic argument we should reinstate the suppressed proposition, and state the syllogism in full. Sophists often take recourse to enthymemes to delude their opponents.

The fallacy of Many Questions (Plures Interrogationes).

We may here name another class of fallacies known as the fallacy of **Many Questions** (Plures Interrogationes). It occurs when one answer is demanded to several questions contained in one sentence. Thus if anyone asks, Have you left off beating your father? no single answer, whether 'yes' or 'no', can be given by

the ordinary person. If he answers 'yes', it implies that once he used to beat his father; if he answers 'no', it implies that he beats his father even now. Similarly it frequently happens that no single answer can be given to such questions as, Where did you hide the goods you stole last night? Have you given up drinking? Have you cast your horns? etc. Sophists often asked such questions to place their opponents in a difficulty. Lawyers often use these questions in cross-examination. In inductive reasoning we often commit a fallacy when we suppose that a cause which produces an effect is the only cause of the event. Thus when we suppose that death has but one cause, or that a particular disease has but one explanation, or that hunger can be appeased only by one kind of food, we commit such a fallacy.

Semi-logical Fallacies

Semi-logical fallacies arise from the ambiguity of language.

Under this head we discuss all those fallacies which arise from ambiguity of language. Some of these are really fallacies of definition, since when terms are not clearly defined they are often used ambiguously. Other fallacies of ambiguity are due to the ambiguous construction of propositions. Fallacies which are due to the ambiguity of language are called by Aristotle fallacies **in dictione.** They are Equivocation, Composition, Division, Figure of speech, Amphiboly and Accent. Besides these, Accident and Secundum quid, which Aristotle includes under the class of fallacies extra dictionem, are also regarded as fallacies due to the ambiguity of language. All these fallacies, and some others, are included by Mill in

'fallacies of confusion'. Let us now explain these fallacies one by one.

1. **Aequivocatio** or **Homonymia.**—Equivocation occurs through the use of words capable of two or more meanings. We use equivocal terms only when we fail to define them. In a syllogistic argument, the middle term or one of the extremes may be ambiguous, and then we have the fallacy of four terms. The following argument is an example given by the old logicians, in which the middle term is ambiguous:—"The end of a thing is its perfection; death is the end of life; therefore death is the perfection of life." 'End' in the major premise means 'aim', and in the minor premise it means 'termination'. The following is another example: "Knowledge is power; perception is knowledge; therefore perception is power." Here the middle term 'knowledge' is used ambiguously in the premises. In one case 'knowledge' means knowledge in general, in the other it means a particular species of it. Words often change their meaning, and owing to the use of such words the fallacy of equivocation often occurs. Thus, 'publication' once meant communication to others, but 'publish' now means to write and print. So 'utter' meant to give out, but it now means to give out of the mouth in words. The meaning of a word should be decided by its current use, not by its etymological sense. The term 'scarcity of money' is often used ambiguously. It may mean either scarcity of currency or scarcity of capital. 'Government' may mean the system of laws, or the persons who are entrusted with the carrying out of those laws.

Fallacy of Equivocation.

'Nature' may mean physical nature, instinct, reason etc. Puns are logically instances of this fallacy. Joseph gives the following example of equivocation: "Men who have recovered are well, the sick man has recovered, therefore the sick man is well." Here the minor term is ambiguous. In the minor premise 'the sick man' means the man who *was* sick, in the conclusion it means the man who *is* sick. The following is an example in which the major term is ambiguous or equivocal: No courageous creature flies; the eagle is a courageous creature; therefore the eagle does not fly. Here the major term 'fly' in the major premise means to flee, while in the conclusion it means to move through the air with wings.

The fallacy of Figure of Speech (Figura Dictionis).

2. The Fallacy of **Figure of speech** (Figura Dictionis or Sophism).—This consists in supposing that words which are similar in form are similar in meaning. It is wrong to suppose that poeta ('a poet') is in the feminine gender because most Latin words with the same termination are so. Mill is wrong in arguing that because what people actually hear is audible, and what people actually see is visible, therefore what people actually desire is desirable. 'Desirable' means that which ought to be desired, not what is actually desired. The meaning of 'desirable' is not parallel to the meaning of the words 'audible' and 'visible'. The most important fallacies of this class arise from the use of paronymous terms or conjugate words. Fallacies of figure of speech are often called fallacies of paronymous terms. Different parts of speech derived from the same root are not always similar in meaning. Thus artist, artisan, artful are not

similar in meaning. Similarly pity and pitiful, presume and presumption, apprehend and apprehension, design and designer, image and imaginary, faith and faithful are not similar in meaning, though the words are derived from the same root. What is imaginary is unreal, but the image of an object, whether of a flower or of a stone or of a house, is real. An artful man need not be an artist. To apprehend means to perceive or understand, but apprehension often means fear or dread. These fallacies also are offences against definition. When paronymous terms are used in syllogisms, we have the fallacy of four terms. Mill gives the following examples of such fallacies :— Murder should be punished with death ; This man is a murderer ; therefore He deserves to die. Here we proceed on the assumption that to commit murder and to be a murderer who deserves death are equivalent expressions. Projectors are unfit to be trusted ; This man has formed a project ; therefore He is unfit to be trusted. A sophist will try to make men believe that a projector is simply one who forms a project, but 'projector' also has the meaning of a promoter of bogus companies. Welton and Monahan give the following example to illustrate the fallacy of figure of speech : What a man walks on he tramples on ; This man walks on the whole day ; therefore He tramples on the whole day. The expression 'walks on' does not mean the same thing in the two premises.

3. The fallacy of **Accident** (Secundum Quid).— Aristotle made a distinction between the fallacy of **Accident** and **Secundum Quid.** But Secundum Quid is regarded by modern logicians as not different from the

The fallacy of Accident explained and illustrated.

fallacy of Accident. The following examples illustrate the fallacy of Accident in the Aristotelian sense (fallacia accidentis), which arises when a predication which can be correctly made of a given subject is made of any of the 'accidents' of that subject: (*a*) This dog is yours, This dog is a father, therefore This dog is your father. (*b*) Do you know Coriscus? Yes. Do you know the man approaching you with his face muffled? No. But he is Coriscus, and you said you knew him. (A man approaching with his face muffled need not be Coriscus, because it is an accidental circumstance.) (*c*) Six is few, and thirty-six is six times six, therefore thirty-six is few.

The three forms of the fallacy of Accident or Secundum Quid.

We have already remarked that what Aristotelians called Secundum Quid is now called the fallacy of Accident. Such fallacies may have three forms:—(*a*) We may pass from an abstract general rule to a particular concrete case. The fallacy is then technically known as 'fallacia **a dicto simpliciter ad dictum secundum quid.**' The following are examples of this fallacy: Employment of labour is beneficial to the community, therefore unemployed workmen may wisely be set to do work of an entirely useless character, merely to find them employment. Similarly to argue from What man has done, man can do, to the conclusion that What Newton or Shakespeare has done you can do, illustrates this fallacy. From the proverb 'a rolling stone gathers no moss', if we conclude that every commercial traveller must starve, we

commit this fallacy. The following example also illustrates this fallacy: Water is liquid, Ice is water, therefore Ice is liquid. In the minor premise 'water' means condensed water. (*b*) We may reason from a concrete special case to an abstract rule. The fallacy is then technically known as 'fallacia **a dicto secundum quid ad dictum simpliciter.**' The following example is given by Welton and Monahan: What you bought yesterday you ate to-day; you bought raw meat yesterday, therefore you ate raw meat to-day. It is not made clear whether rawness is regarded in the major premise as a relevant circumstance, but it is assumed to be relevant in the conclusion. If, finding that a particular Indian is dishonest, we conclude that all Indians are dishonest, we commit this fallacy. (*c*) We may argue from one special case to another special case. The fallacy is then technically known as 'fallacia **a dicto secundum quid ad dictum secundum alterum quid.**' To argue from the assertion that to take life in sport is cruel, to the conclusion that to eat flesh from which life has been taken by others is to show a cruel disposition, illustrates this fallacy. The following example also illustrates this fallacy: To inflict pain an another is wrong; the surgeon in performing an operation inflicts pain on another; therefore the surgeon does something wrong. A master asked his servant to roast a stork. The servant was prevailed upon by his sweetheart to cut off one of its legs and give it to her. When the master at dinner-time asked what had become of one of the legs of the stork, the servant answered that storks had but one leg. The master, desiring to confute the servant before punishing

him, took him to a place where many storks were standing. They were all standing on one leg, as they are accustomed to do. The master shouted, and they put down their other leg and flew away. On this the servant said, "You did not shout at the dinner table; if you had done so, the roasted stork would have put down its other leg." In this case the fallacy consists in the fact that the servant argued that what was true of a living stork was also true of a roasted stork. This is clearly a fallacy of accident.

4. The fallacy of **Composition** and **Division** (Compositio and Divisio). The fallacy of composition and division occurs when we join together things which ought to be kept separate or when we separate those which ought to be kept conjoined. In other words, the fallacy of composition occurs when the same term is distributive in the premises and collective in the conclusion, and the fallacy of division occurs when the same term is collective in the premises but is distributive in the conclusion. These fallacies also are committed when the middle term is taken collectively in one of the premises and distributively in the other. Thus the fallacy of composition and the fallacy of division are the converse of each other. The following argument of Mill is a glaring example of the fallacy of composition:—

The meaning of the fallacy of Composition and Division.

"No reason can be given why the general happiness is desirable except that each person, as far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not

Fallacy of Composition illustrated.

only all the proof which the case admits of, but all which it is possible to require, that happiness is a good: that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons." If A, B, C each desire the happiness of each, it does not follow that each of them will desire the happiness of all. If it is admitted that various subjects ought to be taught in schools for the welfare of the community, it does not follow that all the subjects should be taught to each member of the community. When a debauchee argues that since he can commit particular acts of intemperance without ruining his health, he can commit all these acts successively without any detriment, he commits the fallacy of composition. Again when a spendthrift argues that because he can buy this piano or that book-case or that ring or that table, therefore he can buy all these things together, he commits the fallacy of composition. Aristotle gives the following example of the fallacy of composition: Two and three are even and odd: two and three are five; therefore five is even and odd. The following are examples of the fallacy of division, in which the middle term is collective in the major premise but distributive in the minor:—All the books on the table weigh ten tons, This is a book on the table, therefore This book weighs ten tons. All the angles of a triangle are equal to two right angles, ABC is an angle of a triangle, therefore ABC is equal to two right angles. When a miser argues that because he cannot buy these books and that watch and this table and that picture all at once, therefore he cannot buy any of them,

The fallacy of Division illustrated.

he commits the fallacy of division. Thus we find that failure to distinguish between disjunctive and conjunctive propositions may give rise to the fallacy either of composition or of division. The following argument also illustrates the fallacy of division :—Three and five are (together) four and four (together); but neither three nor five is four; therefore three and five together are not four and four together.

Fallacies incidental to ambiguous construction of propositions.

As a result of ambiguity in the construction of propositions, there may be two classes of fallacies (recognised by Aristotle), *viz.* (1) Amphiboly (amphibolia), (2) the fallacy of Accent (accentus). The fallacies treated of in the previous paragraph arise from the ambiguous use of terms, while the fallacies of amphiboly and accent are incidental to the ambiguous construction of propositions.

The fallacy of Amphiboly explained and illustrated.

1. **Amphiboly** (Amphibolia).—This fallacy is committed when a proposition becomes liable to misinterpretation through its ambiguous construction. The following oracle given to Pyrrhus is an example in point: "Pyrrhus the Romans shall, I say, subdue." This may mean either that Pyrrhus shall subdue the Romans or that the Romans shall subdue Pyrrhus. The following is another classical example of this fallacy: "The Duke yet lives that Henry shall depose" (witch's prophecy in King Henry VI). This proposition may mean either that the Duke shall depose Henry, or that Henry shall depose the Duke. The following line of W. R. Spencer

furnishes another good example of this fallacy: "The noble hound the wolf hath slain." This fallacy occurs in modern English chiefly when words are not used in their proper order. The following are examples: "The first photograph is that of a 14-pound pike taken in a back-yard from the top of a step-ladder;" "a lady (through circumstances) wishes to let part of her well-furnished house;" "the Territorial band played the hymns as well as the church organ." These fallacies can be got rid of if sentences are properly constructed.

The fallacy of Accent explained and illustrated.

2. The **fallacy of Accent** (Accentus).—In Greek the same word, if differently accentuated, had a different meaning. So the fallacy of accent was recognised by Aristotle. Sometimes the meaning of a proposition becomes distorted when emphasis is laid wrongly upon some word which should not be emphasised. Thus the proposition, Thou shalt not bear false witness against thy neighbour, may mean that you may bear false witness against those who are not your neighbour, if emphasis is laid upon the word 'neighbour'. De Morgan rightly points out that accents, gesture and manner often make the difference between irony, sarcasm and ordinary assertion. The father spoke to his sons saying, "Saddle me the ass," and the sons saddled the father. (I Kings 13, 27). We need not provide more examples of this fallacy.

END OF PART I